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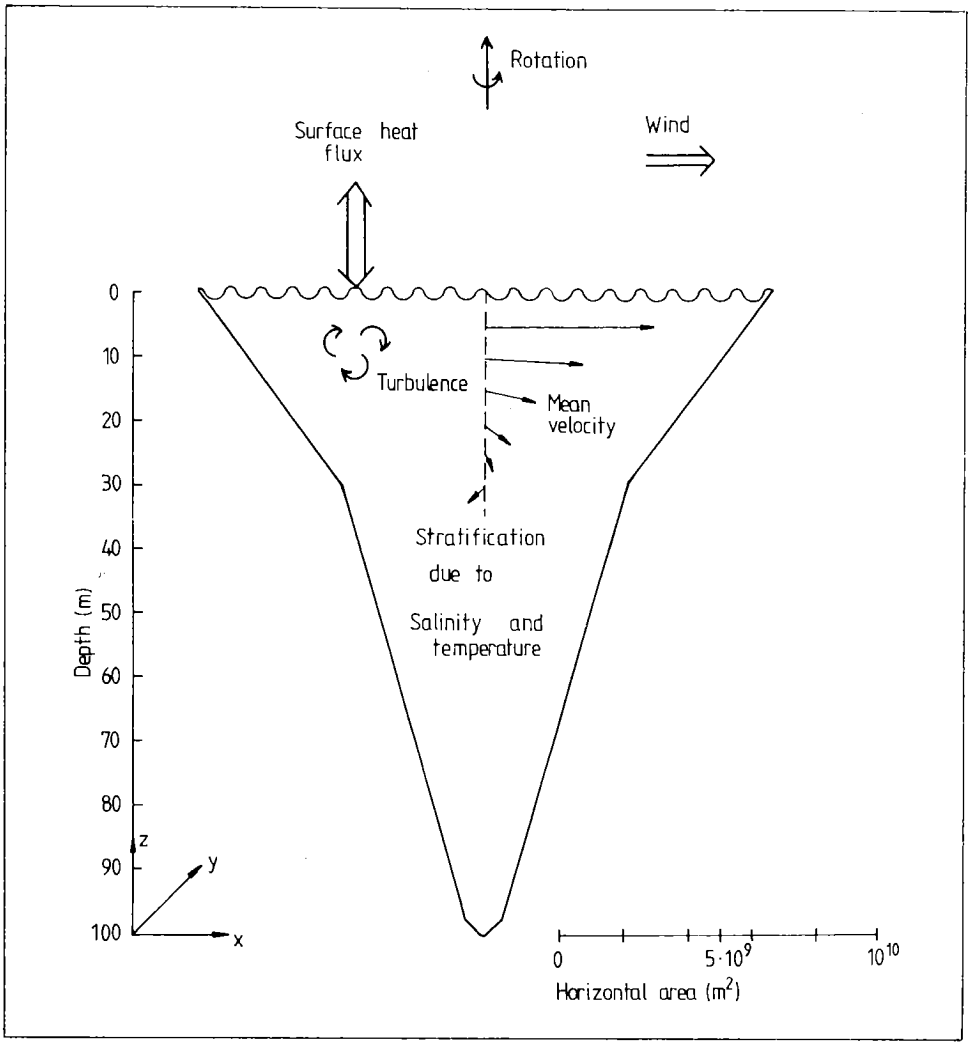

Vertical mixing and restratification in the Bay of Bothnia during cooling

by Anders Omstedt and Jörgen Sahlberg

Norrköping 1982

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SMHIs tryckeri, Norrköping 1982

FOREWORD

The Winter Navigation Research Board presents its report no. 35. The report describes an advanced mathematical model by which the cooling rate and the changes of vertical temperature profiles in enclosed basins can be calculated using meteorological parameters. The model has been adjusted to the Bay of Bothnia and tested against actual vertical temperature measurements during an autumn cooling period. The results presented are very promising.

The study is a part of a thermodynamic ice project aiming at the forecasting of water cooling, ice formation, growth and decay.

The Winter Navigation Research Board expresses its thanks to the authors and those who have assisted them in their work.

Norrköping and Helsinki, March 1982

Kaj Janérus Jan-Erik Jansson

ABSTRACT

Autumn cooling in the Bay of Bothnia provides an opportunity for studying wind mixing, convection and restratification below the temperature for maximum density. Vertical temperature profiles for a 52 day period are analysed for the case of cooling of brackish sea water around the temperature of maximum density. A mathematical model, which is based on the conservation equations for momentum, heat and salt in their one-dimensional form and with an equation of state which is linear with respect to salinity but quadratic with respect to temperature, is presented. Turbulent exchange coefficients are calculated with a kinetic energy-dissipation model of turbulence.

Due to the fact that both salinity and temperature effect stratification and that buoyancy flux changes sign at the temperature for maximum density several processes influence the cooling rate. The mathematical model describes these and the general development of the temperature profiles in a most satisfactory way.

SAMMANFATTNING

Höstavkylningen av Bottenvikens vattenmassa ger möjlighet till studier av vindblandning och konvektion för temperaturer runt temperaturen för maximal täthet. Under en 52-dagars period 1979 mättes vattentemperaturen på 22 olika djup. Dessa data har dygnsmedelvärdesbildats och analyserats. En matematisk modell presenteras här. Den är baserad på en-dimensionella konservationsekvationer för momentum, värme och salt tillsammans med en tillståndsekvation som är linjärt beroende på salthalt och kvadratiskt på temperatur. De turbulenta utbyteskoefficienterna beräknas med en turbulensmodell (k-ε modell).

Den matematiska modellen beskriver förändringarna hos de mätta temperaturprofilerna på ett tillfredsställande sätt.

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1. INTRODUCTION

Under the joint Swedish-Finnish winter navigation research programme the Swedish Meteorological and Hydrological Institute (SMHI) is participating with a programme in which the interaction between atmosphere, ice and sea is studied. One project within this programme treats cooling and freezing in the Bay and Sea of Bothnia. The time of the year when ice starts to form is very variable due to differences in meteorological forcing from year to year, figure 1. It is therefore of great interest for shipping and icebreaking service to get information about changes in sea surface temperatures during autumn cooling as a first information on how soon ice will form.

A simple thermodynamic model for the large scale cooling in the Bay of Bothnia is now used at SMHI, Sahlberg and Törnevik (1980). The model, however, just treats the whole Bay of Bothnia as one layer and therefore neglect the importance of varying temperatures with depth, which are of particular importance before ice formation. The purpose of this paper is to present a model which could calculate the vertical variations in the temperatures during cooling of brackish sea water, that means water with less salinity than $24.7^{o}/oo$, down to the freezing point.

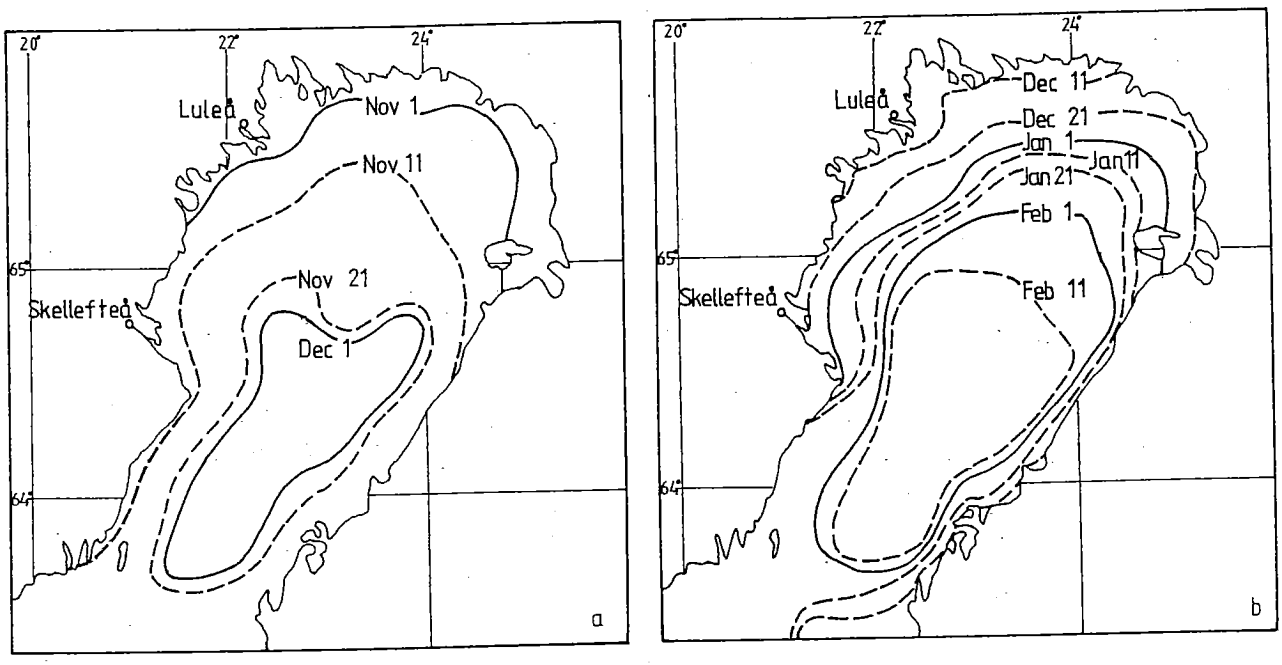

Figure 1 *The charts show the dates for the first (a) and latest day (b) of ice formation in the Bay of Bothnia, SMHI and FIMR (1982).*

In water with a salinity less than $24.7^{o}/oo$, vertical convection continues until the temperature for maximum density ($T_{\rho m}$) is reached, figure 2. Further cooling causes a change in sign of buoyancy flux and creates a restratification which increases the cooling rate.

Many attempts have been made to predict changes in the sea surface layer due to different meteorological conditions. One dimensional models have often been used as temperature and salinity often varies more along vertical axis than horizontal. A discussion of different kinds of one dimensional models for the upper ocean is made by Niiler and Kraus (1977).

The modelling of brackish sea water during autumn and winter cooling has received little attention in the literature, but the temperatures for maximum density and freezing have been considered frequently and these equations are well documented.

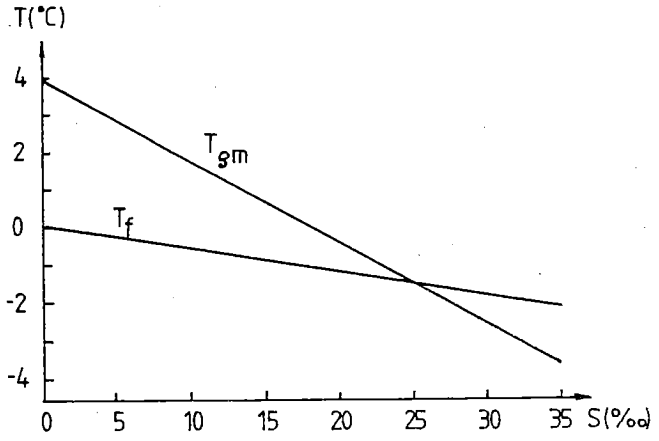

Figure 2 The temperature for maximum density ($T_{\rho m}$) and the temperature for freezing point (T_f).

Several other studies provide useful information for the problem addressed. To be mentioned are the studies of salinity effects in the ocean mixed layer by Miller (1976) and the numerical simulation of thermohaline convection by Delnore (1980). Miller (1976) showed that the most significant effects produced by the inclusion of salinity are the reduction of the deepening rate and the corresponding change in the heating characteristics of the mixed layer.

Delnore (1980) also pointed out the necessity of including gradient information for both temperature and salinity in modelling the diurnal heating cycle of the ocean.

It is also noted that the non linearity and pressure dependence in the equation of state for cooling of deep lakes have been discussed by Farmer and Carmack (1981).

One main problem in modelling the upper layer of the sea is the formulation of the turbulence processes. A wide spectrum of turbu-

lence models is now available. At present a two-equation model, with one equation for the turbulent kinetic energy and the other for the dissipation of turbulent kinetic energy, is most widely tested one for hydraulic flow problems, Rodi (1980).

This kind of turbulence model has also been used for modelling the seasonal thermocline in lakes, in studying the structure of the turbulent Ekman layer, Svensson (1978, 1979) and calculating the turbulent boundary layer under drifting sea ice, Svensson (1981).

The present study extends this turbulent model to the Bay of Bothnia, where the stratification depends mainly on temperature and salinity. The purpose is to analyse measured temperatures for a 52 days cooling period from October 23 to December 13 1979 and to develop a model for autumn cooling.

2. GENERAL DESCRIPTION

The Bay of Bothnia is the northern extension of the Baltic. Climatically it is situated in the northern part of the westerlies and therefore the weather is influenced by the meandering polar front and the disturbances on it which during late autumn could cause strong winds. During late autumn and winter the low pressure systems normally pass south of the area and cold air masses cover the whole bay.

The Bay of Bothnia has a typical estuary circulation with lighter fresh water from rivers mixed with the under-lying saltier water. The water exchange with the Sea of Bothnia goes through the Northern Quark. The amount of saltier water going from the Sea of Bothnia to the Bay of Bothnia is about 400 $km^3 year^{-1}$ and the amount of less saltier water leaving the Bay of Bothnia is about 500 $km^3 year^{-1}$. In late autumn and in winter the water is two layered. The depth of the upper layer varies between 30-50 m and has a salinity of 3.0-3.5°/oo, while the bottom layer salinity is in the range of 4.0-4.5°/oo. Tidal effects are negligible, but water level variations mainly due to wind and air pressure exist with amplitudes in the order of one meter. Maximum water depth is 126 m, while the mean depth is 41 m and the Northern Quark sill depth 20 m. A typical residence time for the water in the Bay of Bothnia is about 3 years which is a rather slow process compared

with autumn cooling. It might thus be expected that cooling in the main basin is mostly influenced by the heat exchange through the air-water interface.

During autumn the cooling causes horizontal temperature gradients due to water depth variations close to the coast. In the main basin however the horizontal temperature gradients are less pronounced.

The internal hydrographic response in a system like the Bay of Bothnia is mainly baroclinic in a narrow coastal region, while the response in the main basin is essentially barotropic, Walin (1972). This means that in the main basin, temperature and salinity surfaces are horizontal and one dimensional approach could be possible.

For the cooling rate the heat loss at the air-sea interface and the mixing depth are of most importance. The mixing depth depends mainly on wind mixing, convection and restratification when cooling has passed the temperature of maximum density. These processes can be well described by one dimensional boundary layer theory.

3. MODEL EQUATIONS

3.1 Basic assumptions

The study will restrict attention to horizontally homogeneous flows which means that terms containing gradients in the horizontal plane are neglected. It is further assumed that there is no mean vertical velocity. Short wave radiation is known to penetrate the water body following an exponential decay. During the period considered the solar radiation is however of minor importance and will therefore be treated as a surface flux of heat. Gravitational effects are assumed to follow the Boussinesq approximation. A schematic representation of the mathematical model as applied to the Bay of Bothnia is given in figure 3. The variation of horizontal area versus depth is in accordance with the Bay of Bothnia. This is of fundamental importance when modelling changes in the total heat content, Svensson (1978).

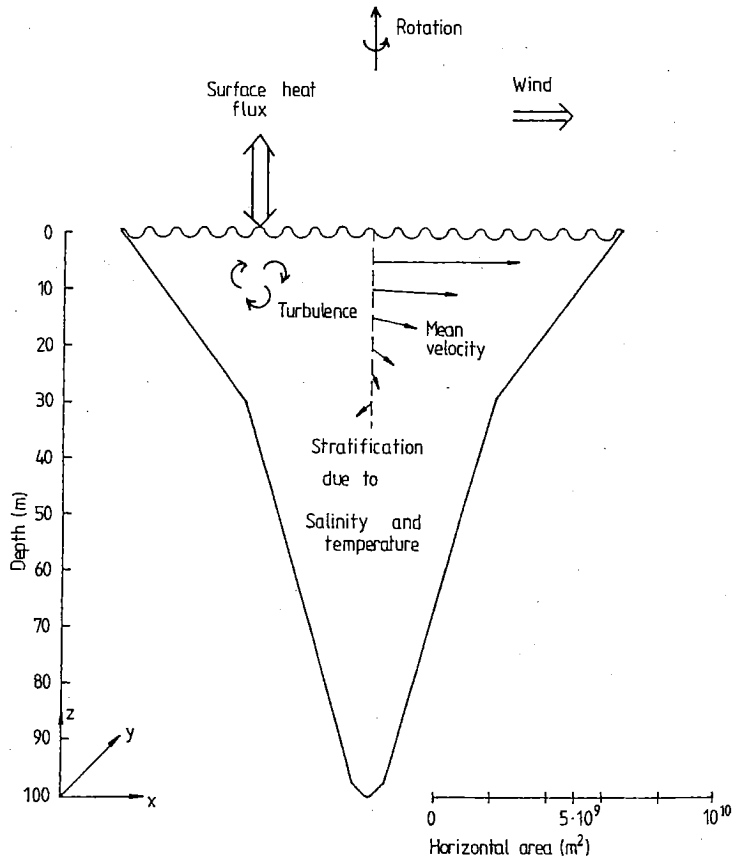

Figure 3 Schematic representation of the one dimensional model for the Bay of Bothnia.

3.2 Mean flow equations

Primarily the temperature distribution is of interest but a few more variables are needed to describe the problem. Salinity is one of them and another is the velocity distribution which, needs to be considered since turbulence is to a large extent produced by shear. With the assumptions made the equations for the variables read:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial Z}\left(-\overline{w\theta}+\kappa\frac{\partial T}{\partial Z}\right), \tag{1}$$

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial Z}\left(-\overline{ws}+\kappa_s\frac{\partial S}{\partial Z}\right), \tag{2}$$

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial Z}\left(-\overline{wu}+\nu\frac{\partial U}{\partial Z}\right)+fV, \tag{3}$$

$$\frac{\partial V}{\partial t} = \frac{\partial}{\partial Z}\left(-\overline{wv}+\nu\frac{\partial V}{\partial Z}\right)-fU, \tag{4}$$

where Z is the vertical space coordinate, positive upwards, t time coordinate, f Coriolis parameter, U and V mean velocities in horizontal directions, T mean temperature, S mean salinity, ν molecular kinematic viscosity, κ molecular thermal diffusivity and κ_s molecular diffusivity for salt. The correlation terms $\overline{wu}$, $\overline{wv}$, $\overline{w\theta}$ and $\overline{ws}$ represent Reynold stresses and turbulent transport of heat and salt respectively.

Boundary conditions for the mean flow equations at the surface are specified according to:

$$(-\overline{wu}+\nu\frac{\partial U}{\partial Z}) = \frac{\tau_x(t)}{\rho_o}, \tag{5}$$

$$(-\overline{wv}+\nu\frac{\partial V}{\partial Z}) = \frac{\tau_y(t)}{\rho_o}, \tag{6}$$

$$(-\overline{ws}+\kappa_s\frac{\partial S}{\partial Z}) = 0 , \tag{7}$$

$$(-\overline{w\theta}+\kappa\frac{\partial T}{\partial Z}) = \frac{Q(t)}{\rho_o C_p} , \tag{8}$$

where ρ_o is the water density, $\tau_x(t)$ and $\tau_y(t)$ wind stresses, C_p specific heat of water and $Q(t)$ net heat flux. $Q(t)$ is of course a sum of several processes as sensible heat flux, latent heat flux and radiation fluxes. They will be discussed further in chapter 3.5. The zero flux condition for salinity is an approximation since advection from rivers, precipitation and evaporation in effect generate a non zero flux. At the lower boundary a zero flux condition is used for all variables, the reason being that this boundary is positioned well below the depth of surface influence and the deep water flow due to estuary circulation is neglected.

The mean flow equations however do not form a closed system due to the turbulence correlation terms. An equation for the turbulence is therefore needed, but first the equation of state has to be studied.

3.3 Equation of state

The equation of state for sea water describes the relation between

salinity, temperature, pressure and density. From very accurate measurements in the Baltic in the beginning of this century, Knudsen et al (1901) constructed hydrographical tables for density. These tables are still in use and later measurements have shown that they are in good agreement when salinity is higher than 15°/oo, Kremling (1972). At lower salinities the comparison differs due to input of dissolved river salts into the Baltic and an other clorinity-salinity relation has to be used when determining the density for low salinity water, Miller and Kremling (1976). Pressure effects both the density and the temperature for maximum density. This has been treated by Farmer and Carmack (1981) in a study of cooling in a deep fresh water lake. As a lake cools through the temperature for maximum density, the thermal expansion coefficient becomes critically dependent on pressure and a phase where stratification is mainly pressure sensitive is reached. The importance of the pressure effects increases rapidly with depth. In the Bay of Bothnia salinity gradient however inhibits convection to deeper layer why pressure effects have been neglected.

As the relationship between salinity, temperature and density is nonlinear and rather complicated a simplified equation of state is used. This equation should reproduce the almost quadratic relation between temperature and density around the point of maximum density and also the linear dependence on salinity, figure 4.

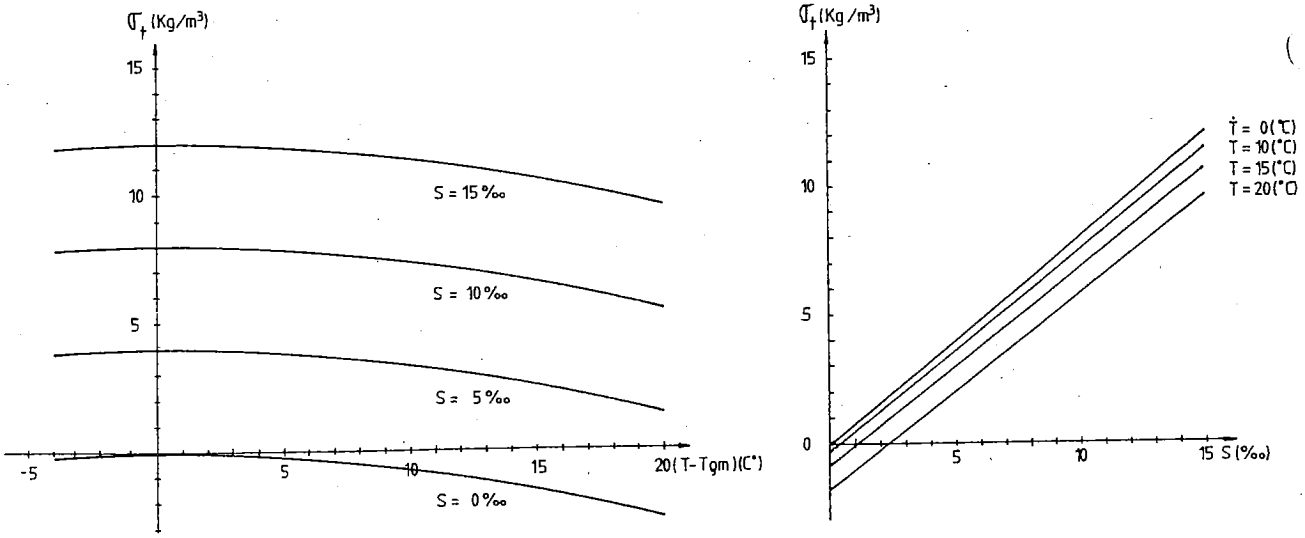

Figure 4 The density of sea water as a function of temperature and salinity.

The equation of state used in this study reads:

$$\rho = \rho_o(1-\alpha(T-T_{\rho m})^2+\beta s) \qquad (9)$$

where α and β are constants and $T_{\rho m}$ is the temperature of maximum density. The constants α and β are chosen with respect to the temperature and salinity interval under consideration. In this study they are set to $5.57 \cdot 10^{-6}$ and $8.13 \cdot 10^{-4}$ respectively. The temperature of maximum density, $T_{\rho m}$, is strictly a function of salinity and pressure, Caldwell (1978). In this study $T_{\rho m}$ is however set to a constant value of $3.2^{o}C$ which is within $\pm$ $0.1^{o}C$ accuracy for the mixed layer under consideration. As density variations are small a more useful density unit is Sigma-t (σ_t) defined as

$$\sigma_t = \rho - 1000.$$

in SI-units.

A comparison between densities calculated from equation (9) and from Knudsen et al (1901) modified with a chlorinity-salinity relation according to Miller and Kremling (1976), is given in figure 5. The accuracy of equation (9) within the studied interval is $\pm$ 0.03 (kg/m^3).

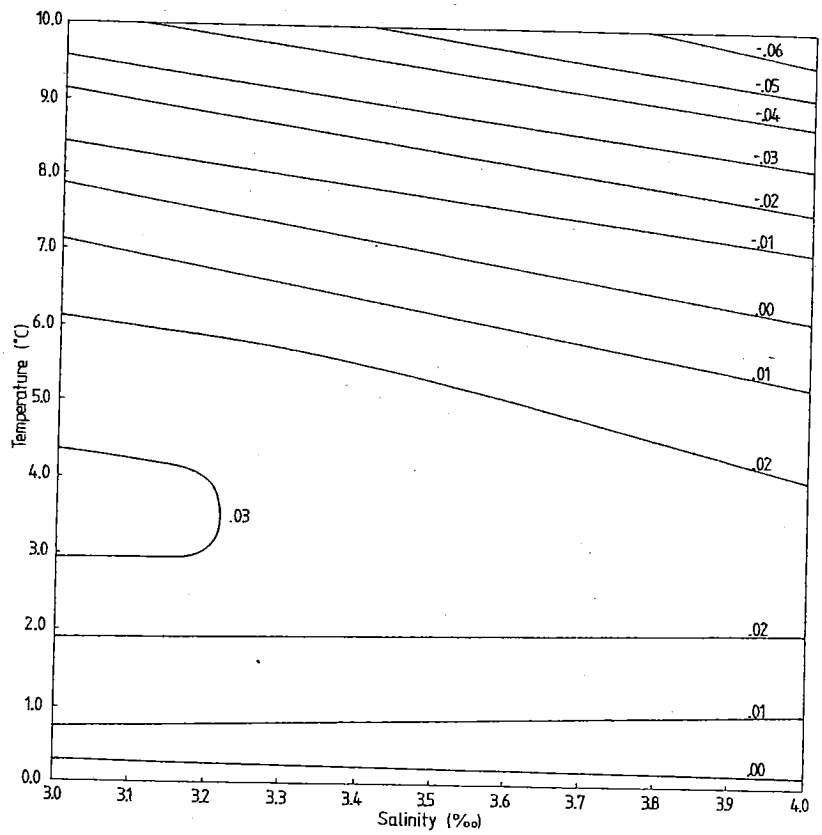

Figure 5 *Error diagram in σ_t-units for sea water density according to Knudsen et al (1901) modified with a chlorinity-salinity relation according to Miller and Kremling (1976) and according to eq. (9).*

The relative importance of the salinity stratification grows when the temperature for maximum density is reached and even very small changes in salinity can cause the density structure to look quite different. The relative importance between the salinity and temperature on density becomes:

$$\frac{\partial\rho}{\partial T}/\frac{\partial\rho}{\partial s} = 2\ \frac{\alpha}{\beta}(T-T_{\rho m})$$

which, with coefficients according to this study, means that wiggles in temperature of $1^{o}C$ around the temperature for maximum density could be stabilized by salinity variations of about $0.01^{o}/oo$. Small salinity variations in the upper sea surface layer are therefore of most importance for the density structure.

3.4 Turbulence equations

A fundamental process in mixed layer dynamics is the conversions of energy. Turbulent kinetic energy can be converted into potential energy. A budget equation for k, the turbulent kinetic energy, is needed. With the introduction of a kinematic eddy viscosity, ν_T, a modelled form of the k-equation, see Launder and Spalding (1972) for details, reads:

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial Z}\left(\frac{\nu_T}{\sigma_k}\frac{\partial k}{\partial Z}\right)+\nu_T\left(\left(\frac{\partial U}{\partial Z}\right)^2+\left(\frac{\partial V}{\partial Z}\right)^2\right)+\frac{g}{\rho}\nu_T\frac{\partial\rho}{\partial Z} - \varepsilon, \qquad (10)$$

where σ_k is a Prandtl/Schmidt number, g gravitational acceleration and ε dissipation rate of k. Another advantage of solving an equation for k is that a velocity scale, $k^{1/2}$, is available for the determination of ν_T. From physical reasoning and dimensional analysis it is expected that ν_T is the product of a velocity scale and a length scale. Given the velocity scale a length scale is thus needed. If once again dimensional analysis is employed it can be shown that a length scale, l, can be obtained from k and ε as:

$$l \sim k^{3/2}/\varepsilon \qquad (11)$$

Recalling that the velocity scale is $k^{1/2}$, the Prandtl/Kolmogorov relation for ν_T is obtained:

$$\nu_T = C_\mu k^2/\varepsilon, \qquad (12)$$

where C_μ is an empirical constant. It remains to formulate an equation for ε if ν_T is to be calculated according to (12).

This equation is the weak point of most turbulence models present up to the date and some scientists argue that a prescribed length scale is as good as any dynamical equation presented. The success met by the ε-equation in predicting a wide range of shear flows, see Rodi (1980) and Singhal and Spalding (1981), does indicate a certain degree of generality and the form of the ε-equation given by these scientists is therefore employed:

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial Z}\left(\frac{\nu_T}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial Z}\right) + C_{1\varepsilon}\nu_T \cdot \frac{\varepsilon}{k}\left(\left(\frac{\partial U}{\partial Z}\right)^2 + \left(\frac{\partial V}{\partial Z}\right)^2\right) + C_{3\varepsilon\rho}\frac{g}{k}\frac{\varepsilon}{k}\nu_T\frac{\partial \rho}{\partial Z} - C_{2\varepsilon}\frac{\varepsilon^2}{k} \qquad (13)$$

Having introduced the eddy-viscosity concept it is now possible to relate the turbulence correlations in the mean flow equations to their appropriate gradients through the following expressions:

$$-\overline{wu} = \nu_T\frac{\partial U}{\partial Z}, \qquad (14)$$

$$-\overline{wv} = \nu_T\frac{\partial V}{\partial Z}, \qquad (15)$$

$$-\overline{w\theta} = \frac{\nu_T}{\sigma_T}\frac{\partial T}{\partial Z}, \qquad (16)$$

$$-\overline{ws} = \frac{\nu_T}{\sigma_S}\frac{\partial S}{\partial Z}, \qquad (17)$$

where σ_T and σ_S are turbulent Prandtl/Schmidt numbers for temperature and salinity respectively. At present σ_T and σ_S are assumed to be constant, and equal to one, even if a dependence on stratification is known to exist.

The empirical constants appearing in the turbulence model are treated as being universal. For complex flows, i.e. recirculating, curved, etc, this universality may be questioned but for boundary flows the standard values, used in the present study and shown in Table 1, may be used with confidence.

Equations (1), (2), (3), (4), (9), (10), (12), (13), (14), (15), (16) and (17) form a closed system and thus constitute the formu-

lation of the mathematical model. This set of equations, in their finite difference form, were integrated forward in time with a time step of one hour, using an implicit scheme and a standard tridiagonal matrix algorithm. A complete derivation of the finite difference equations could be found in a paper by Svensson (1978). In chapter 4.3 different eddy viscosity assumptions and their consequence for cooling are discussed.

C_μ	$C_{1\varepsilon}$	$C_{2\varepsilon}$	$C_{3\varepsilon}$	σ_k	σ_ε
0.09	1.44	1.92	0.8	1.0	1.3

Table 1 Constants in turbulence model

3.5 Processes at the air-sea interface

The processes at the air-sea interface should in general be treated as a three dimensional boundary layer problem, which means that both horizontal and vertical variations in the atmosphere should be considered. From atmospherical boundary layer experiments, bulk formulas have been determined which parameterize the heat and momentum exchange at the air-sea interface from a fixed level above the sea surface. With these bulk formulas the boundary layer is reduced to two dimensions and only horizontal variations in the atmosphere had to be considered.

In this work the cooling of the whole Bay of Bothnia is studied in a one dimensional sense. The horizontal variations in the atmosphere is therefore just treated by taking the input data for the bulk formulas as areal mean values over the whole basin. This is discussed in chapter 5.2.

The surface heat budget equation is

$$Q = Q_S + Q_L + Q_C + Q_E \qquad (18)$$

where Q_S is the net short wave radiation, Q_L net long wave radiation, Q_C sensible heat flux and Q_E latent heat flux. During late autumn and in winter time the net short wave radiation (Q_S) is of

minor importance. For a complete description Q_S is here taken into account and it becomes, see also Bodin (1979)

$$Q_S = (1-\alpha')T_u Q_o \cos Z'(T_R - A_W) \prod_{i=1}^{3} (1-N_i(1-T_i)) \qquad (19)$$

where α' is the sea surface albedo, T_u the air turbididty, Q_o solar constant ($1395 Wm^{-2}$), Z' zenith angle, T_R transmission function, A_W absorption function, N_i amount of clouds of category i = 1, 2, 3, T_i cloud function. The surface albedo is chosen as a function of the zenith angle according to Thompson (1979). Due to lack of turbidity data T_u is constant and equal to 1. The transmission function is

$$T_R = 1.041 - 0.16(\text{Sec}Z')^{0.5}$$

given by Kondratyev (1969). The absorption function is

$$A_W = 0.077(U\text{Sec}Z')^{0.3}$$

where U is the amount of precipitable water vapour content in the atmosphere in cm, Mc Donald (1960).

Low, middle or high clouds effect the short wave radiation differently. According to Pandolfo et al (1971) the cloud function is

$$T_i = a_i b_i \text{Sec}Z'$$

As the extracted weather data in this study only include one cloud coverage parameter, the constants a_i and b_i are chosen according to the middle high transmission function with the values of 0.45 and 0.01 respectively.

The net long wave radiation (Q_L) consists of two parts, one going from the sea surface to the atmosphere ($Q_{L\uparrow}$) and one going from the atmosphere to the sea surface ($Q_{L\downarrow}$). The outgoing long wave radiation is

$$Q_{L\uparrow} = \varepsilon_m \sigma T_s^4 \qquad (20)$$

where ε_m is the water emissivity equal to 0.97, σ the Stefan-Bolzmans constant equal to $5.67 \cdot 10^{-8} (Wm^{-2}K^{-4})$.

The major problem in determining the net long wave radiation is to get a proper estimate of the incoming long wave radiation ($Q_{L\downarrow}$). In this study Brunt's formula modified with a cloud factor according to Washington et al (1976) is used. It then reads

$$Q_{L\downarrow} = \sigma T_a^4 (c+d(e_a)^{0.5})(1+hN) \qquad (21)$$

where c, d and h are empirical coefficients equal to 0.67, 0.05 and 0.25 respectively, e_a is the atmospheric water vapour pressure (mb), N the total cloud cover in tenths and T_a the air temperature.

The sensible and latent heat flux formulations are taken from Friehe and Schmitt (1976). They used bulk aerodynamic formulas for the sensible heat flux (Q_C) and the latent heat flux (Q_E) according to

$$Q_C = \rho_a C_p^a C_C |\bar{V}^a| (T_s - T_a) \qquad (22)$$

$$Q_E = L C_E |\bar{V}^a| (Q_s - Q_a) \qquad (23)$$

where ρ_a is the air density, C_p^a specific heat at constant pressure for the air, C_C and C_E heat transfer coefficients, $|\bar{V}^a|$ mean wind speed, L latent heat of evaporation, Q_s and Q_a water vapour densities at the sea surface and the atmosphere respectively. The heat transfer coefficients C_C and C_E are put equal to $1.41 \cdot 10^{-3}$ and $1.32 \cdot 10^{-3}$ respectively. Measurements of Q_s and Q_a are not directly available why a transformation to the water vapour pressure (e_s) and (e_a) was done. Under the assumption that the air close to the sea surface is saturated and has the same temperature as the sea surface, e_s is only a function of the sea surface temperature. The water vapour pressure in the air, e_a, is a function of both air temperature and relative humidity. Based upon humidity data from Bjuröklubb the relative humidity is put constant equal to 90%.

Within above assumptions, the surface heat budget equation (18) can be solved just knowing wind velocity, air temperature, water temperature and cloud amount.

Apart from the surface heat fluxes the wind stress, τ, on the water surface had to be calculated. Generally a quadratic stress law is used

$$\tau = \rho_a C_d |\mathbf{V}^a| \mathbf{V}^a \qquad (24)$$

where $\mathbf{V}^a$ is the wind velocity at 10 meters above sea surface and C_d the wind drag coefficient. In general it is a function of the wind and the stability but in this study a constant drag coefficient, equal to $1.3 \cdot 10^{-3}$, is used.

4. MODEL ILLUSTRATIONS

4.1 Introduction

Before analysing the cooling of the Bay of Bothnia some simplified calculations are presented to illustrate the model with special emphasis on changes in water temperatures.

The presence of salinity in the Bay of Bothnia causes great difference compared with lakes. Neglecting salinity effects the model describes temperature changes in a lake. Note, however, that pressure effects on density and temperature of maximum density are neglected in this paper, see Farmer and Carmack (1981). For constant net heat loss of 200 Wm^{-2}, constant area versus depth and a constant wind of 5 ms^{-1} the solution becomes according to figure 6.

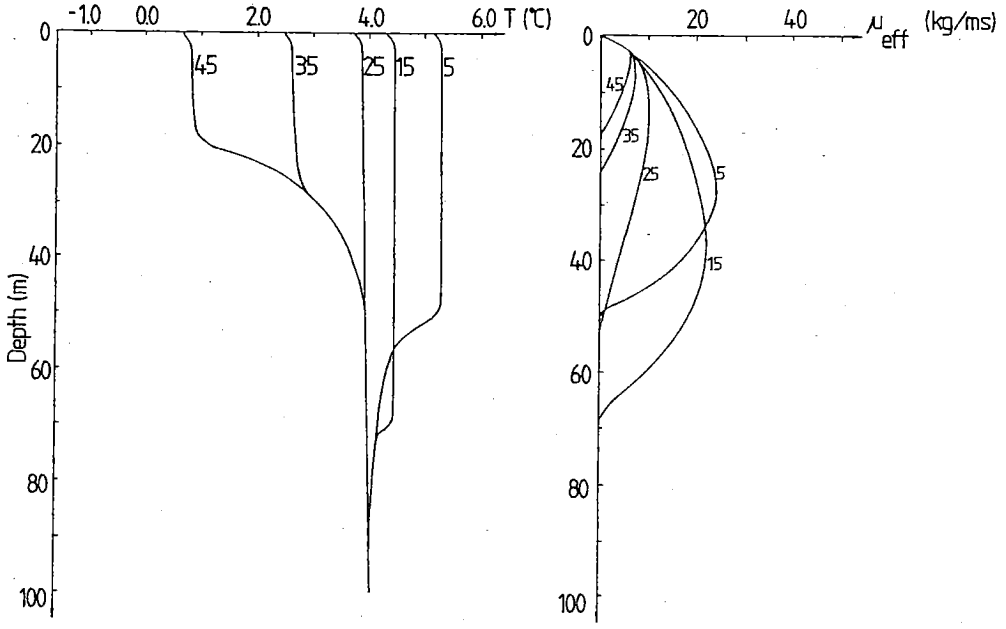

Figure 6 Cooling in a lake during idealized conditions. The wind velocity is 5 ms^{-1} and the net heat loss 200 Wm^{-2}. The numbers at the curves are day numbers.

From the computed temperature profiles one can see how the fresh water mass first is cooled down to the temperature of maximum density, which is close to $4^{o}C$. The whole water column, down to the bottom, reaches this temperature and further cooling leads to restratification. This speeds up the cooling rate of the surface water and cooling down to freezing point can be rather fast, particularly if the wind is weak.

Turbulent exchange in the water is illustrated by the right diagram in figure 6. The dynamic eddy viscosity values (μ_{eff}) represent how effective the turbulent processes are. Large values, due to convection and shear, are calculated before the temperature of maximum density is reached. After this the buoyancy flux changes sign and restratification starts reducing the turbulent exchange.

4.2 Cooling of brackish sea water

If the water is saline some additional processes become important for the cooling rate. In this chapter initial profiles of salinity and temperature is used according to figure 7 and the variation of horizontal area versus depth is in accordance with the Bay of Bothnia.

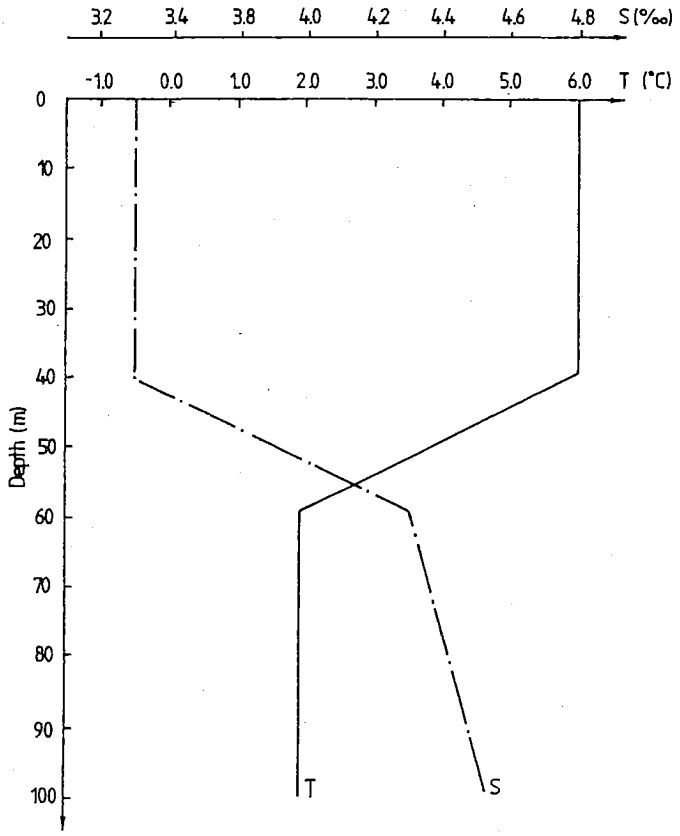

Figure 7 Initial profiles of temperature (T) and salinity (S) for the model illustrations.

Figure 8 illustrates cooling of brackish sea water in a basin corresponding to the Bay of Bothnia with a constant net heat outflow of 200 Wm^{-2} and a wind speed of 5 ms^{-1}. Two most important differencies compared to figure 6 are that the temperature for maximum density is now lowered down to 3.2°C and that the halocline inhibits convection down to the bottom. The salinity profile also reduces the deepening rate of the wind mixed layer which could for example be seen in equation 10, where a stable salinity gradient always works as a sink in the turbulent kinetic energy equation and therefore reduces the turbulent exchange. From figure 8 it is also seen a negative temperature gradient just below the well mixed layer. This is possible as the salinity gradient stabilizes the density profile.

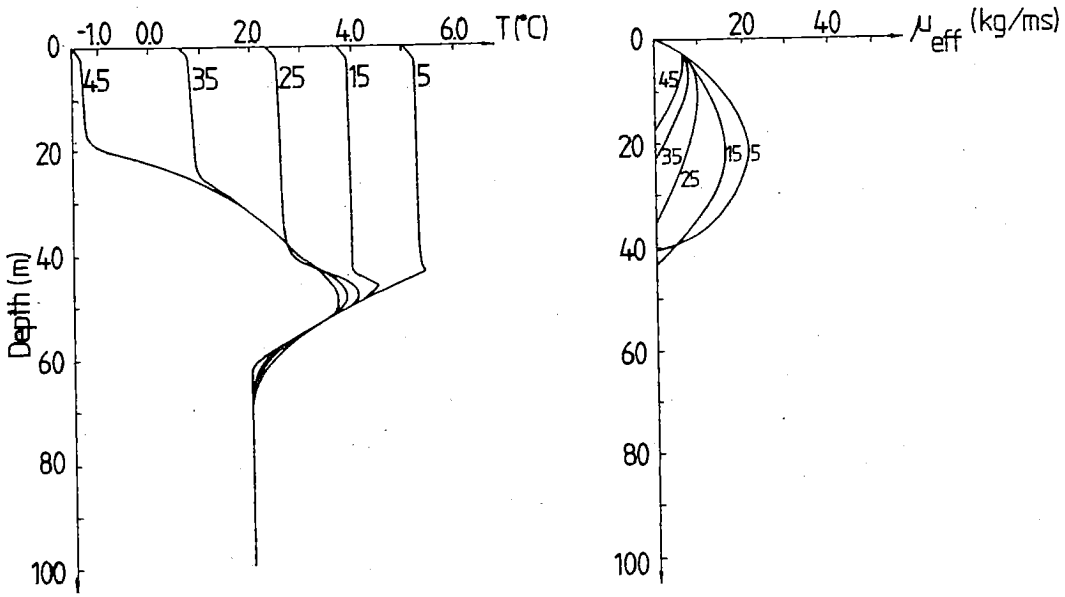

Figure 8 *Cooling of brackish sea water during idealized conditions. The wind velocity is 5 ms^{-1} and the net heat loss 200 Wm^{-2}. The numbers at the curves are day numbers.*

Meteorological forcing is however most changeable. To illustrate the importance of wind mixing on the cooling rate, calculations were made for zero wind speed and wind speeds of 15 ms^{-1}, figure 9. The net heat outflow is as before 200 Wm^{-2}. The figure 9 should be compared with figure 8. Due to the wind, the mixed layer differs

considerably. This highly effects the cooling rate. One could for example see that cooling down to freezing point during zero wind condition takes about 25 days. If the wind speed is 15 ms^{-1}, the freezing temperature is not reached even after 45 days.

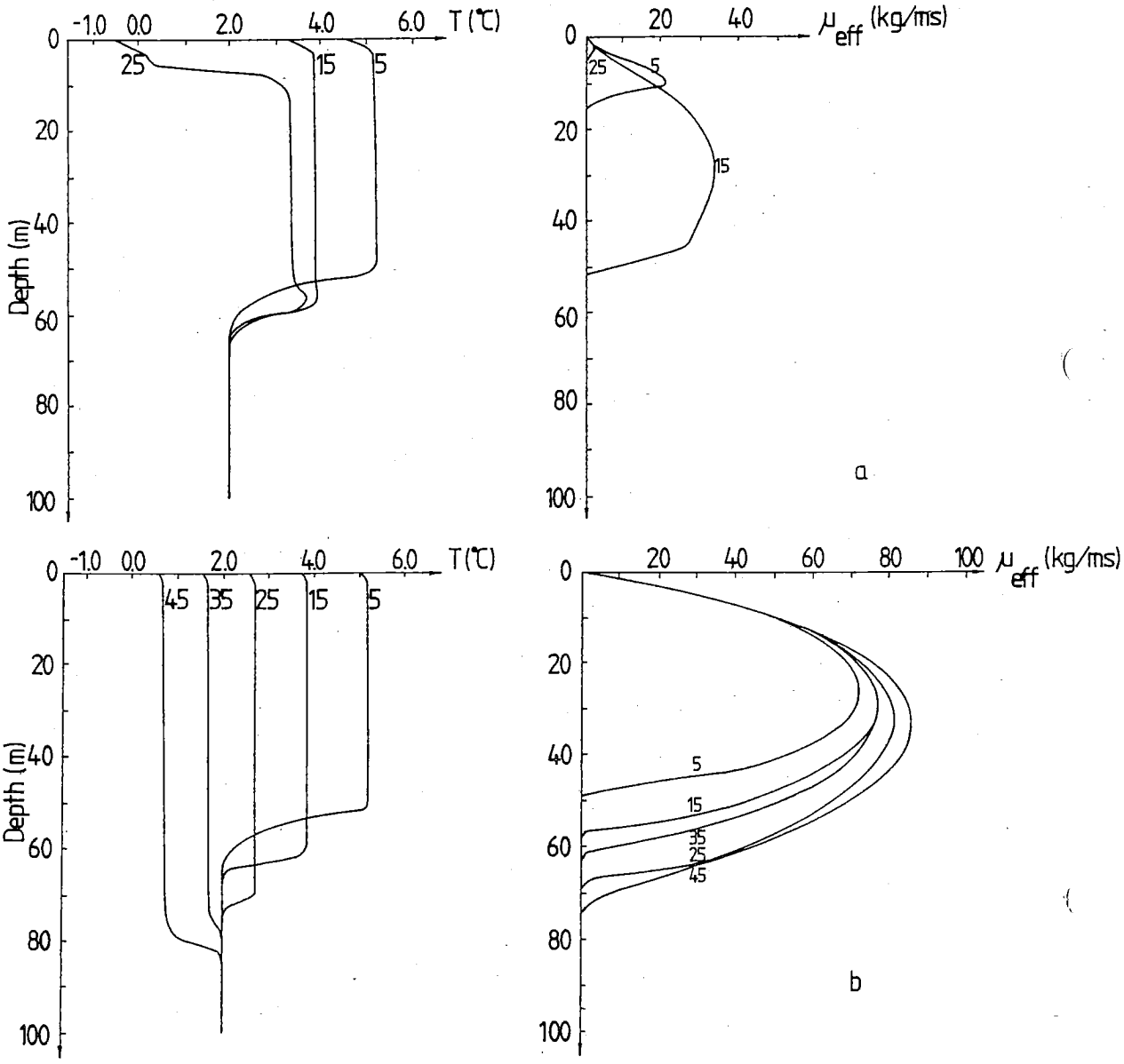

Figure 9 *Cooling of brackish sea water during idealized conditions. In a) the wind velocity is 0 ms^{-1} and in b) the wind velocity is 15 ms^{-1}. The net heat loss is 200 Wm^{-2}. The numbers at the curves are day numbers.*

4.3 Different eddy viscosity assumptions

A main problem in modelling the mixed layer is to represent the turbulent processes in a physical way. One classic approach to

this problem was made by Ekman (1905). Ekman used a constant vertical exchange coefficient when he studied the influence of the earth's rotation on ocean currents. Later work has proposed eddy diffusion coefficients of different forms. For example Madsen (1977) pointed out that an eddy viscosity coefficient which increases linearly from the sea surface is a more realistic model for the wind induced Ekman boundary layer. From observations in the Baltic, Kullenberg (1976) studied the vertical mixing in the surface layer of the sea. In his work just conditions with fairly strong winds and stable stratified waters were treated. During cooling both stable and unstable condition must be considered, and therefore the complete turbulent kinetic energy equation must be solved.

Figure 10 illustrates how important different assumptions on the eddy viscosity are for the cooling rate. In figure 10 a, b and c the net heat outflow is 200 Wm^{-2} and the wind speed is 5 ms^{-1} but different eddy viscosity formulations are used. Figure 10 a illustrates a constant dynamical eddy viscosity coefficient which increases linearly with depth in the upper surface layer, but is constant in the main water column. The calculated temperature profiles are completely unrealistic compared with temperature profiles in the Bay of Bothnia. In figure 10 b also a constant dynamical eddy viscosity coefficient is assumed which increases linearly close to the sea surface and is constant just down through the well mixed layer. The calculated temperature profiles seem a little more realistic but the negative temperature gradient just below the well mixed layer is to exaggerated and the solution also disregards the dynamics of the turbulent processes. In figure 10 c the solution is in accordance with the model presented in this paper.

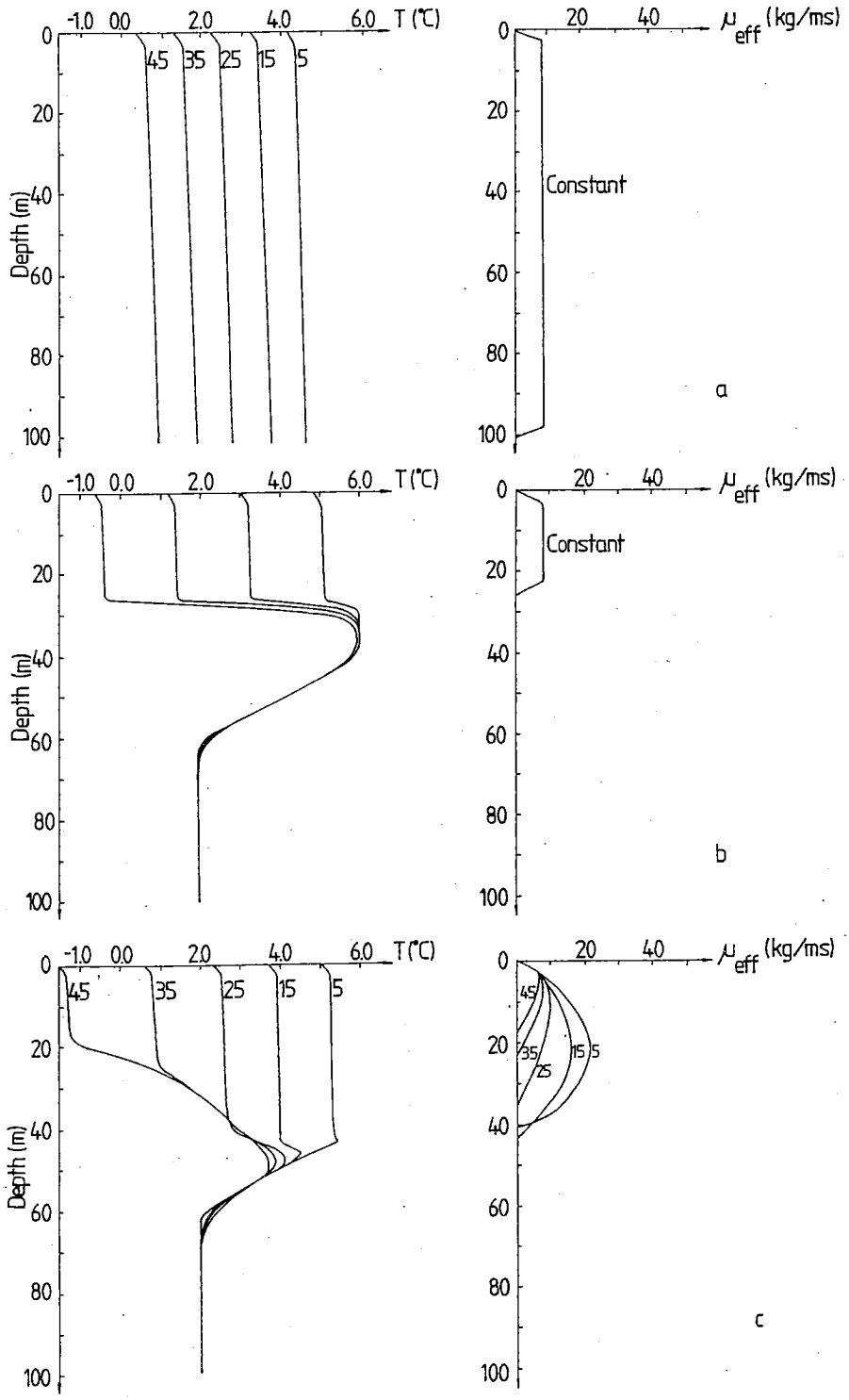

Figure 10 *Cooling of brackish sea water with different turbulent models. In all cases the wind speed is 5 ms^{-1} and the net heat loss is 200Wm^{-2}. The numbers at the curves are day numbers.*

5. MODEL APPLICATION TO THE BAY OF BOTHNIA

5.1 Water temperature data

On October 22, 1979 a water temperature measuring system was placed in the Bay of Bothnia, see figure 11. The system contained two thermistor chains, type Aanderaa, with 11 thermistors in each. The water temperature was measured from a depth of 1 m below the sea surface to a depth of 75 m. The vertical resolution was 2 m for the upper chain and 5 m for the lower chain. Temperatures were measured every 30 minutes and stored on a magnetic tape. The relative accuracy of the data is better than 0.05°C. Due to the risk of ice formation the experiment ended at December 14. The temperature system had then been working for 52 days and fortunately no data were lost.

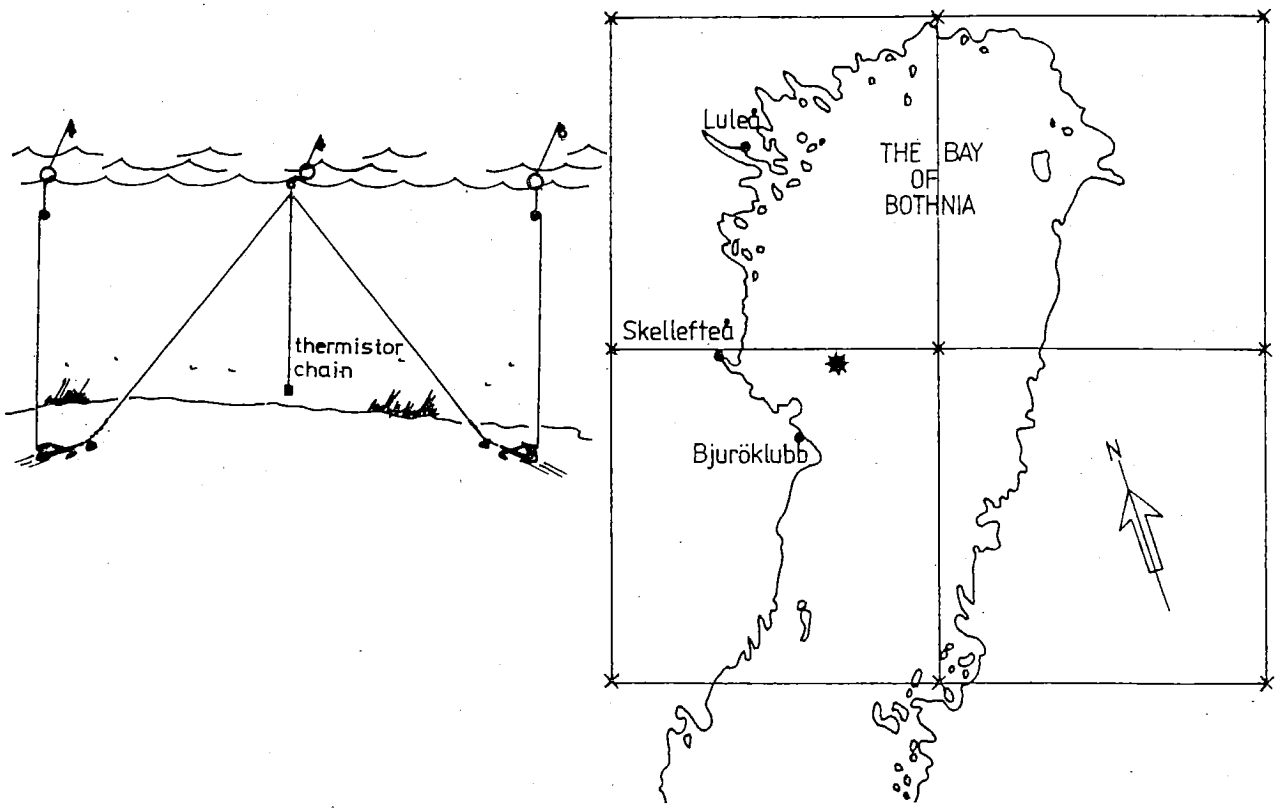

Figure 11 *Map over the Bay of Bothnia with the 150 km meteorological grid given by the crosses. The position of the temperature measurements is indicated by the star. The measuring system is shown schematically beside the map.*

To get a first insight in how the water was cooled during the period, changes in heat content were calculated from observed temperature data. In figure 12 the heat content in the upper 50 meters and in the deeper layer are plotted separately. The upper layer is influenced by meteorological forcing and therefore the heat content decreases during the cooling. The deeper water layer on the other hand shows a slight increase in the heat content. This is probably due to advective currents and could not be predicted by a one-dimensional theory. From figure 12 it is also seen a rather rapid change in heat content between November 8 to 14. This change is seen in all temperature data which indicate that advective currents could be important during this period.

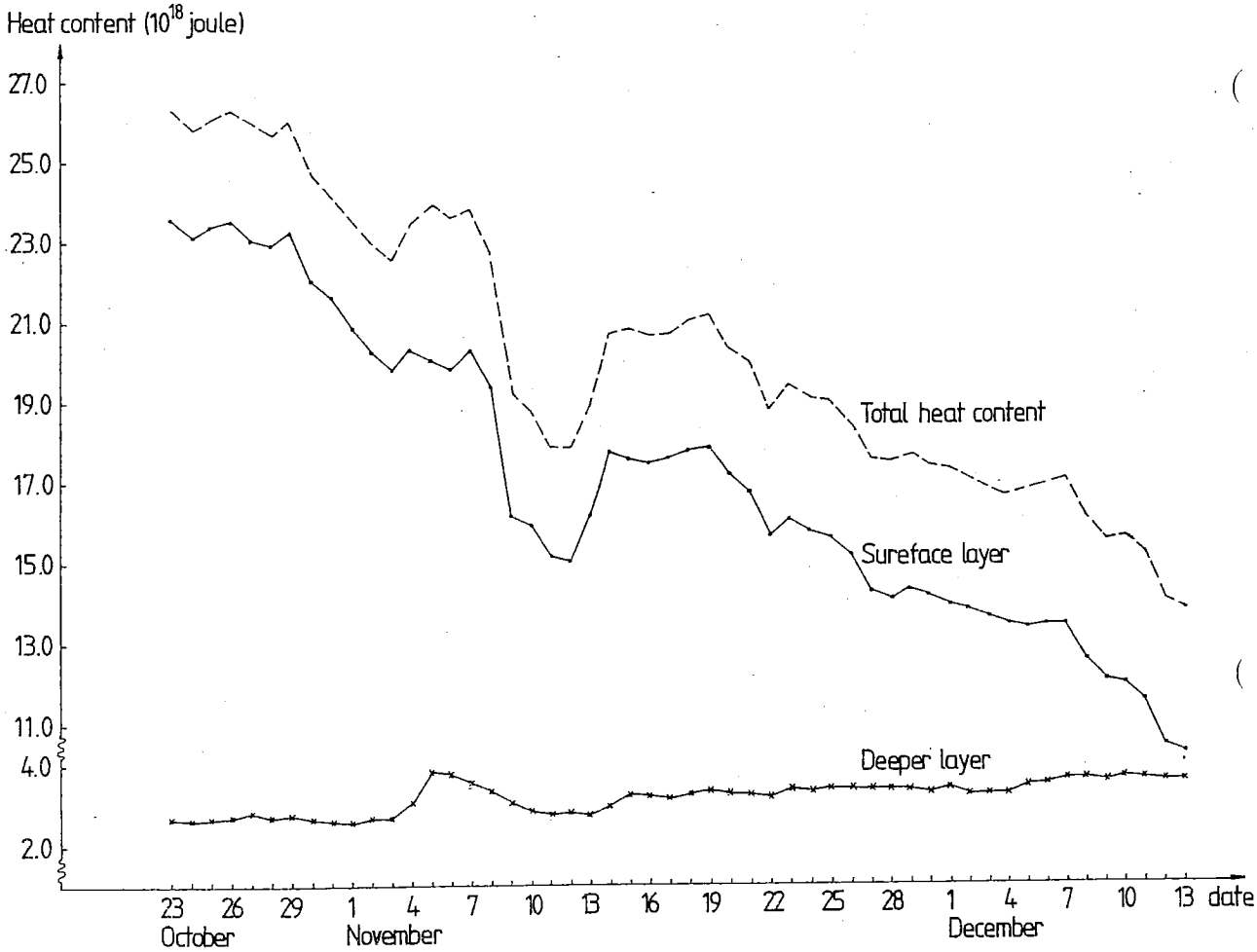

Figure 12 Heat content in the Bay of Bothnia based on the thermistor chain.

In the upper layer of the sea it is also interesting to study how the vertical temperature profiles vary during autumn cooling. In figure 13, the measured temperatures are plotted as 24 hours mean values for the whole period. The figure shows several interesting features. The temperature profiles start with a pronounced thermocline. During cooling a negative temperature gradient is reached.

Further cooling deepens the mixed layer and at the temperature of maximum density the temperatures are almost constant with depth in the surface layer. Passing the temperature of maximum density re-stratification starts in the surface layer and the well mixed depth is reduced.

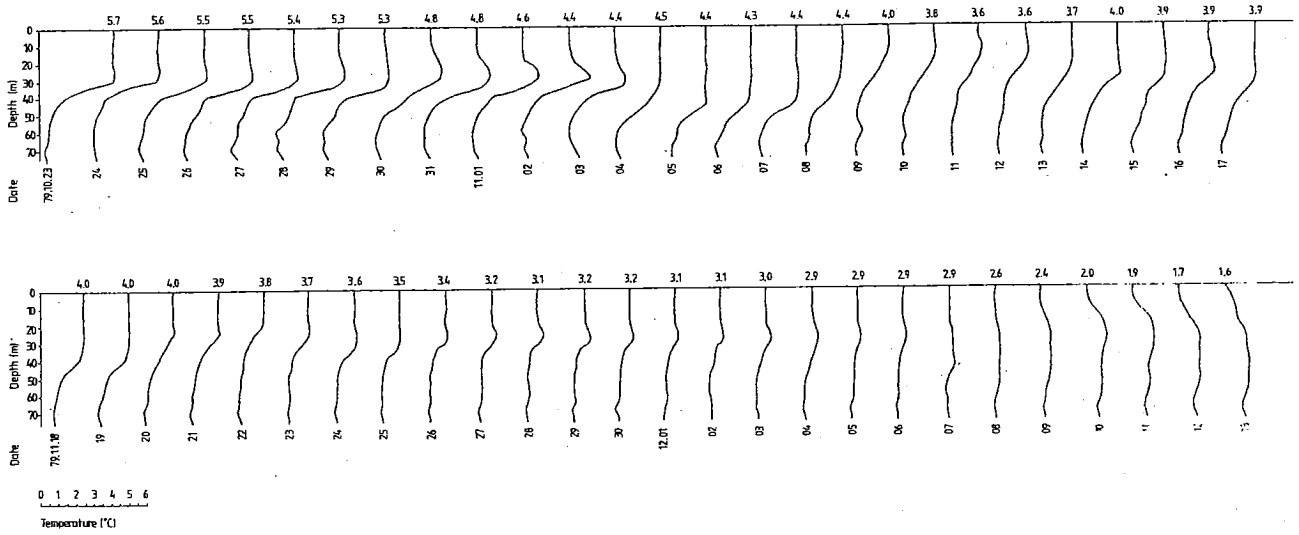

Figure 13 Measured temperature profiles, represented as 24 hour mean values.

5.2 Meteorological data

To calculate the heat and radiation fluxes and the wind stress, weather data have been extracted from analysed synoptic weather charts. This was done every third hour in a 150 km cartesian grid, see figure 11. Air temperature and air surface pressure data were extracted from nine grid points. The cloudiness was extracted as an areal average over the whole Bay of Bothnia. The grid used here is the same as the grids used in the routine weather models. In the future it will therefore be easy to couple the cooling model with a routine weather forecast model. The geostrophic wind speed was calculated from the extracted air pressure data. Together with areal mean temperatures and cloud coverage values net heat flux and wind stress were calculated according to chapter 3.5. Mean

values of the meteorological data are shown in figure 14.

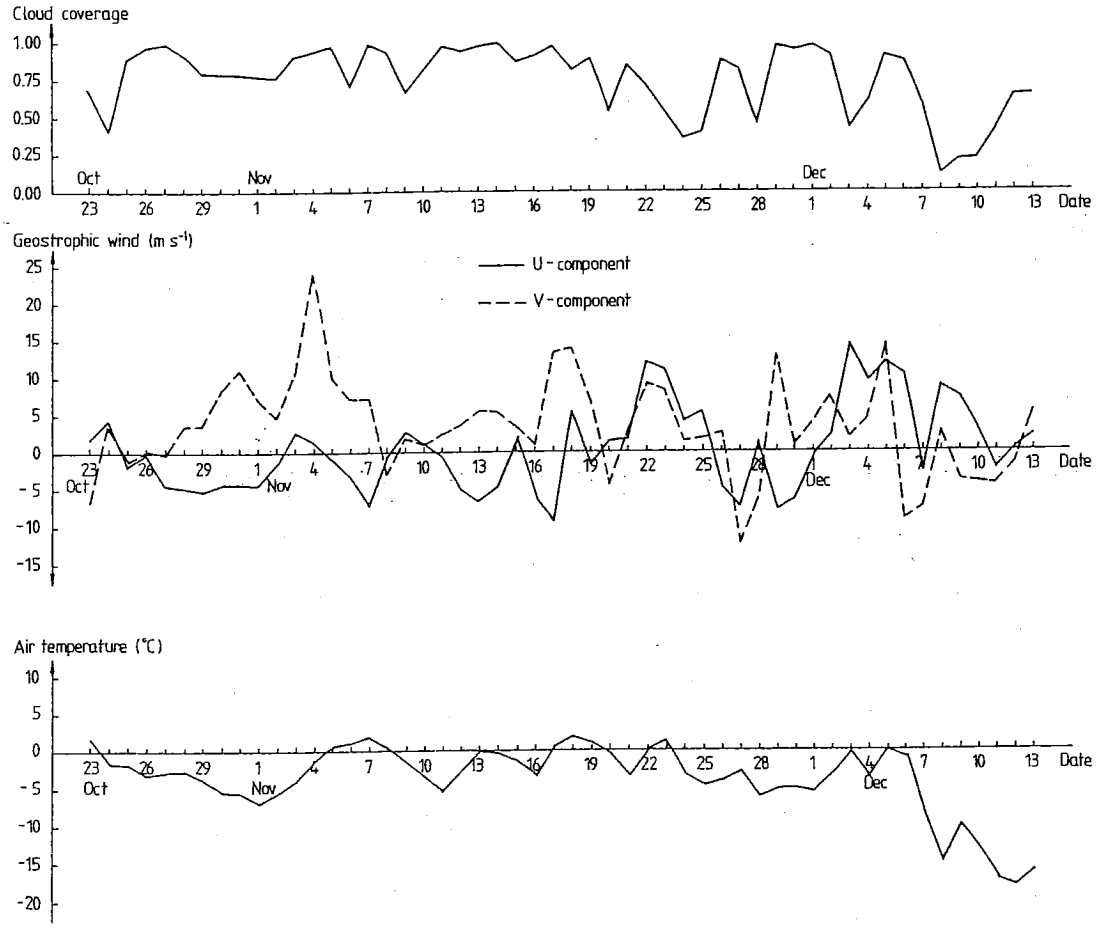

Figure 14 Meteorological data.

5.3 Comparison between calculated and observed temperatures

In this section the results of the field measurements and computations with the mathematical model will be presented. The model calculated the water temperatures with just initial profiles in salinity and temperature and with meteorological input data according to synoptic weather charts. Boundary conditions for the mathematical model were obtained as described in the previous section. Starting profiles in temperature and salinity is shown in figure 15, where the salinity profile is in accordance with an observation made by the Swedish Coast Guard close to the temperature system.

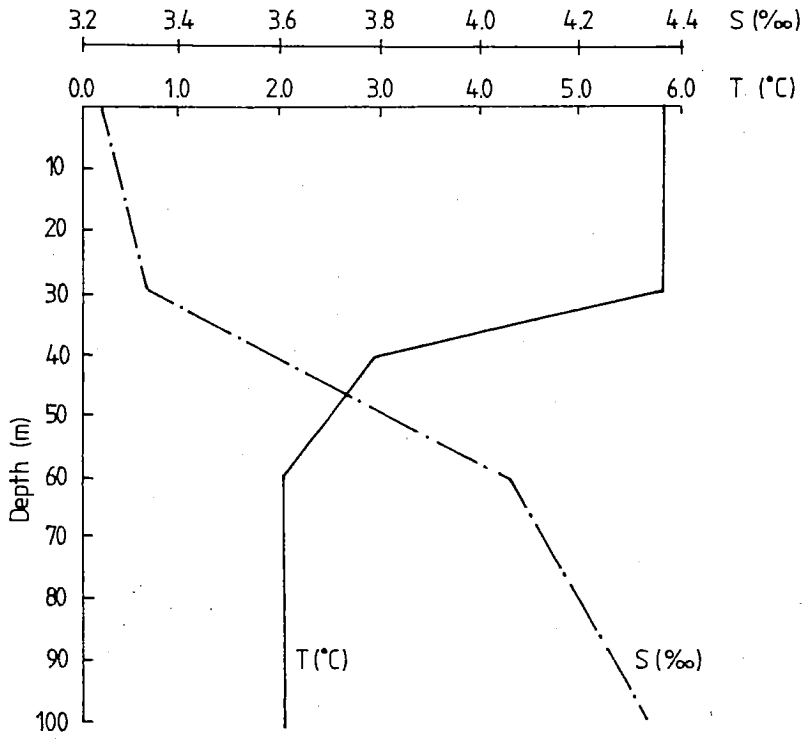

Figure 15 Initial profiles of temperature (T) and salinity (S) based upon measurements from October 23, 1979.

Computed and observed temperature profiles are compared in figures 16 and 17. In general there is a very good agreement between model results and observations. The deepening of the thermocline due to wind mixing and convection and the development of the restratification are well calculated.

The interest is now focused on details of measured and predicted temperature profiles. Figure 18 shows profiles of measured temperature (dashed line) and calculated profiles of temperature (T), salinity (S), density (σ_T) and dynamical turbulent viscosity (μ_{eff}).

The first profile to be examined is the one on October 28. It is clear that both the measured and the predicted profile have a temperature maximum below the well mixed layer due to the combined effect of temperature and salinity gradients which cause stable stratification. During the next few days the model seems to be more sensitive to the increased wind stress and cooling than the measured profiles. In the model the temperature maximum below the well mixed layer is eroded faster than shown in the observed data.

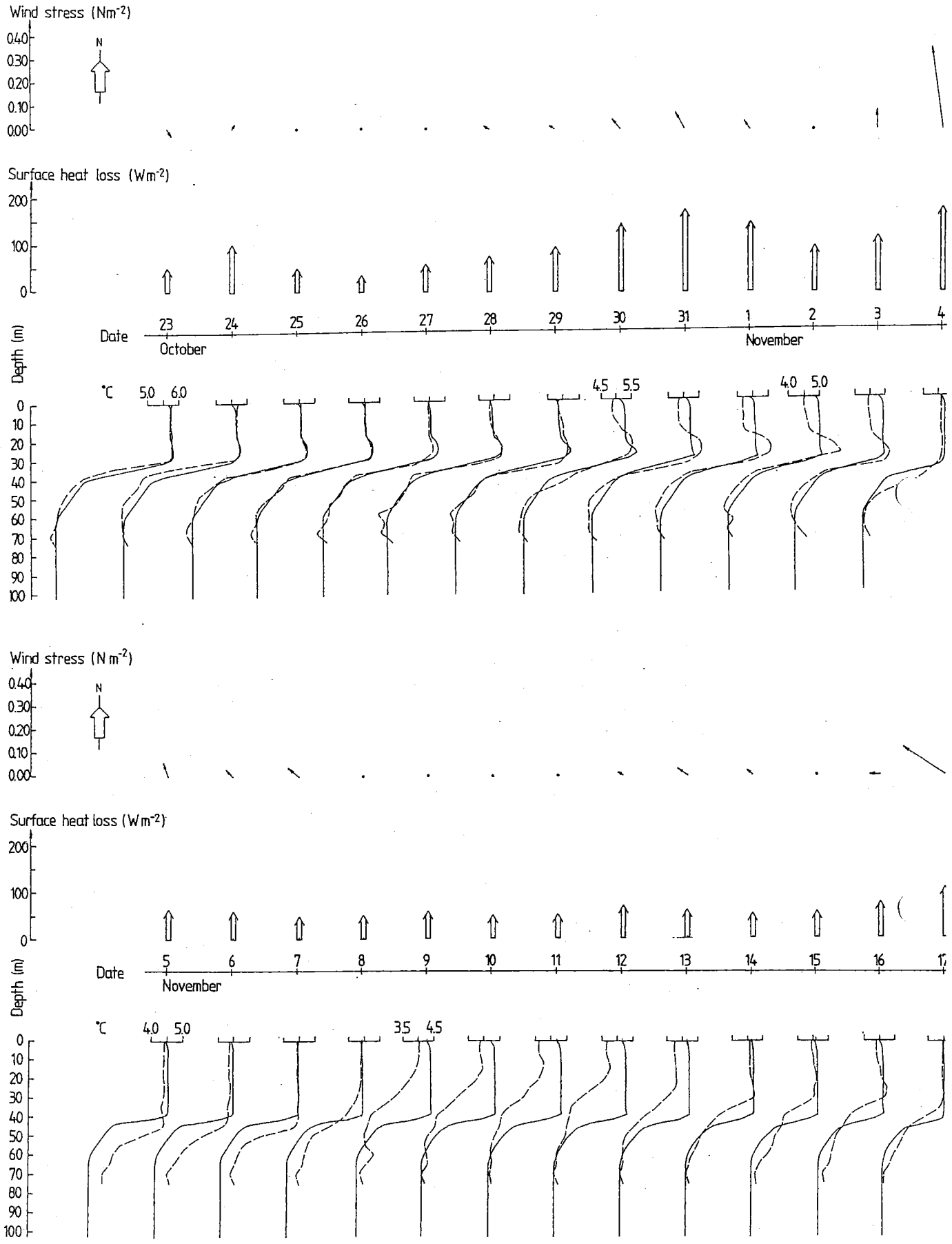

Figure 16 *Calculated wind stresses, heat losses and results of the model run during the first 26 days, compared with observed temperature data (dashed line). All data are 24 hours mean values.*

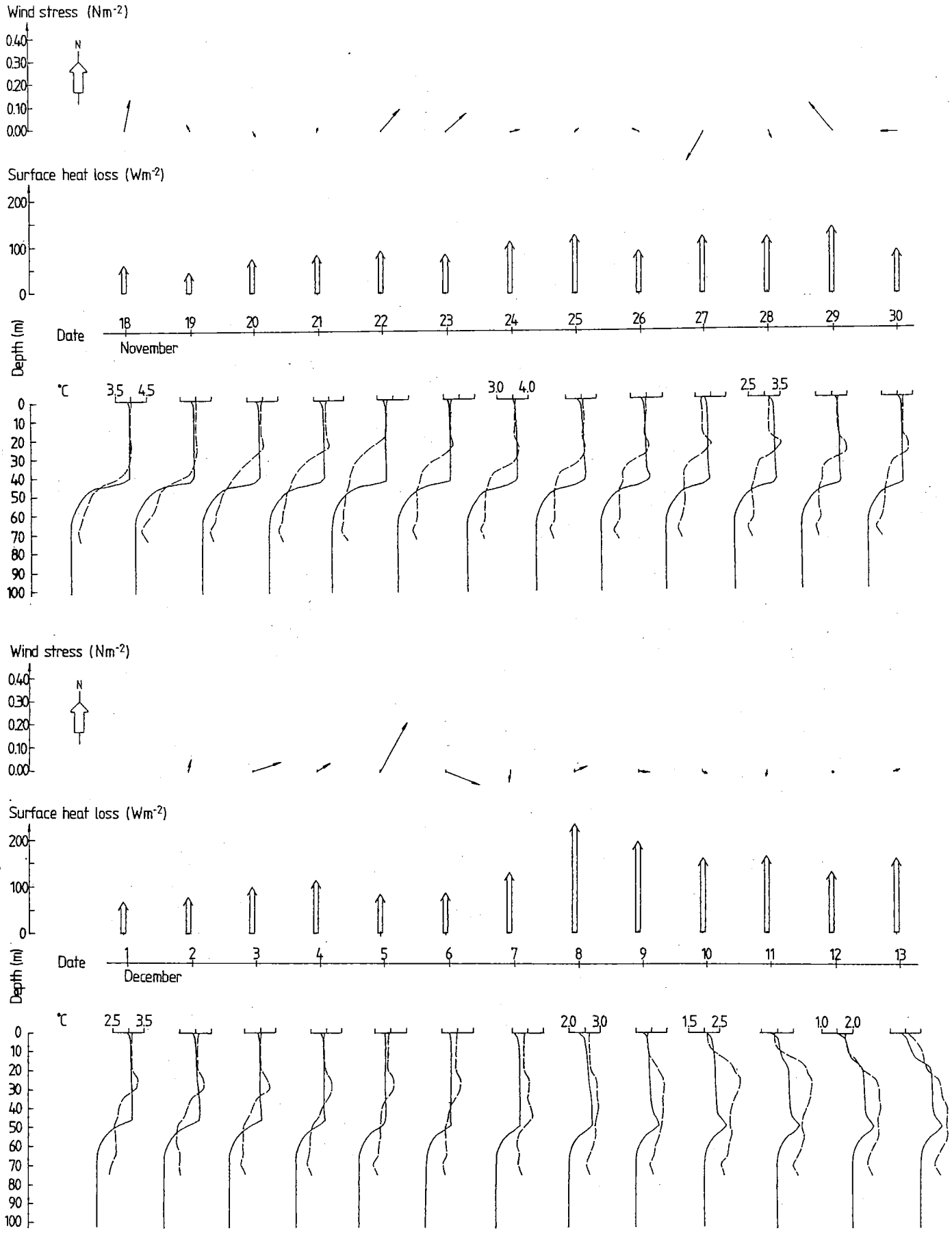

Figure 17 *Calculated wind stresses, heat losses and results of the model run during the last 26 days, compared with observed temperature (dashed line). All data are 24 hours mean values.*

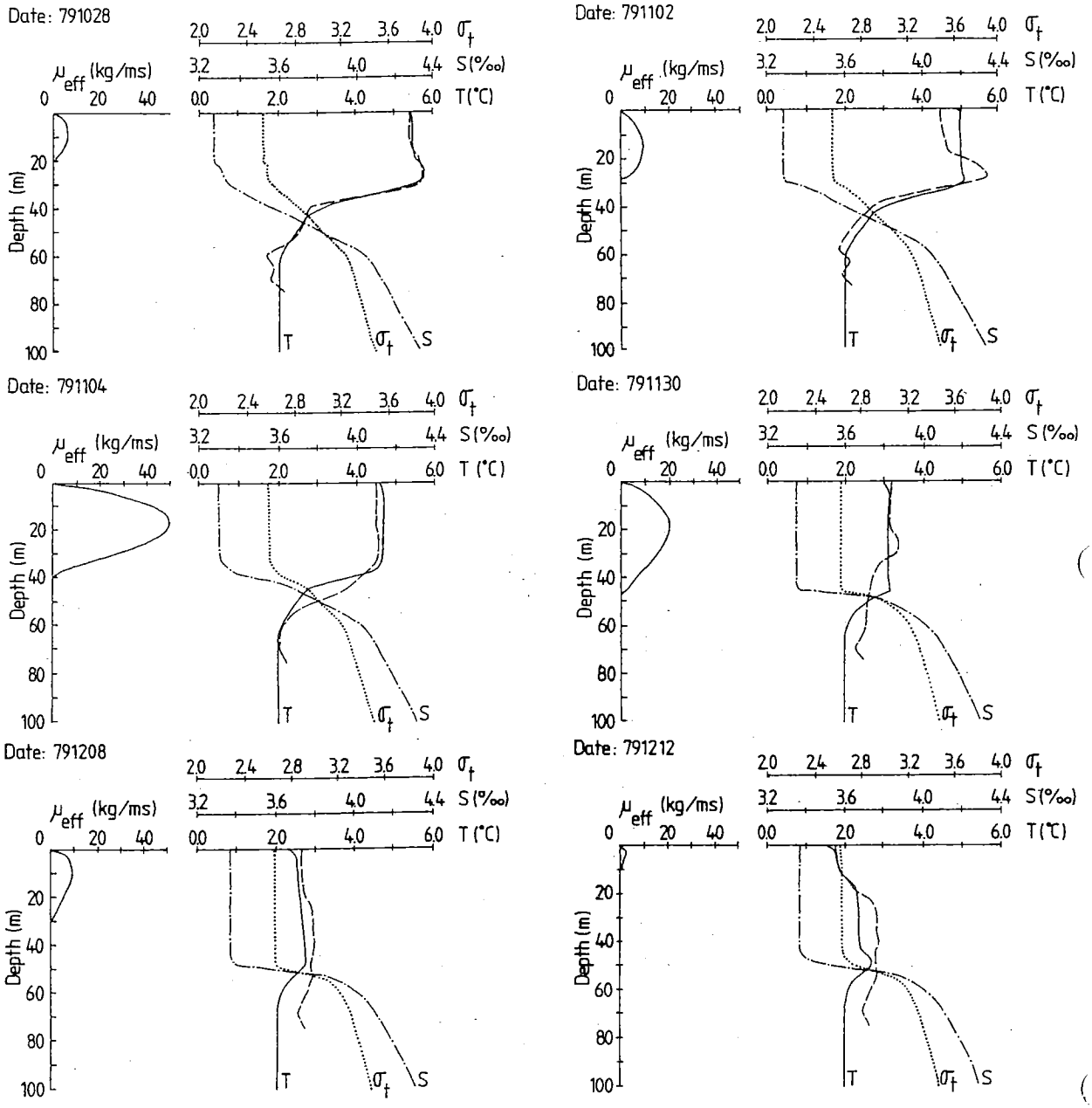

Figure 18 *Model calculations of dynamical eddy viscosity (μ_{eff}), temperature (T), salinity (S) and density (σ_t) for six occasions during the cooling period. The dashed lines are the observed water temperatures. All data are 24 hours mean values.*

As no sinks and sources exist in the salinity equation and as advection is neglected, a much better result cannot be expected.

November 2 the depth of the well mixed layer is over predicted but two days later there is a strong wind and the depth of the mixed layer is again in good agreement with measurements. Details of these two days is found in figure 18.

A less encouraging period starts on November 9 and lasts for about five days. The cause of the disagreement might be traced by noting the rapid change of total heat content of the water column from November 7 to 9. The change is not caused by the surface heat flux. Advective transports are thus the only explanation. In figure 19 the explanation is firmly supported. It is seen that the change in the heat content by net surface heat flux does not correspond to actual change in heat content during the period. Actually, a diagram like this points out periods of strong advective influence, periods when one dimensional models are inadequate. The next date to pay particular attention to is November 30, since the mixed layer temperature now passes the temperature of maximum density, 3.2^{o}C. Further cooling causes restratification. In figure 18 the mixed layer depth is almost 50 m as compared to 20 m on October 28. The temperature of the mixed layer is still correctly predicted even if the measured profile is not as straight as the one predicted. In December both temperature and salinity gradients act to reduce the mixing. December 8 and 12 are displayed in more detail in figure 18. The eddy viscosity profiles show that the fully turbulent layer is much more shallow than what could be concluded from the density profiles. The weak stable stratification due to the temperature is giving this effect. Interesting enough, these conditions are favourable for once again producing a maximum in temperature below the mixed layer. During the last days of the period the maximum is found both in measurements and predictions.

Presumably the examination of the particular profiles has demonstrated that cooling of brackish sea water brings out a number of interesting phenomenon. When judging the performance of the mathematical model one ought to have in mind the importance of salinity on density, that just initial profiles in temperature and salinity are used and also the inevitable uncertainty in the boundary conditions. In light of this the agreement obtained after 52 days of

integration is, in the authors view, most satisfactory.

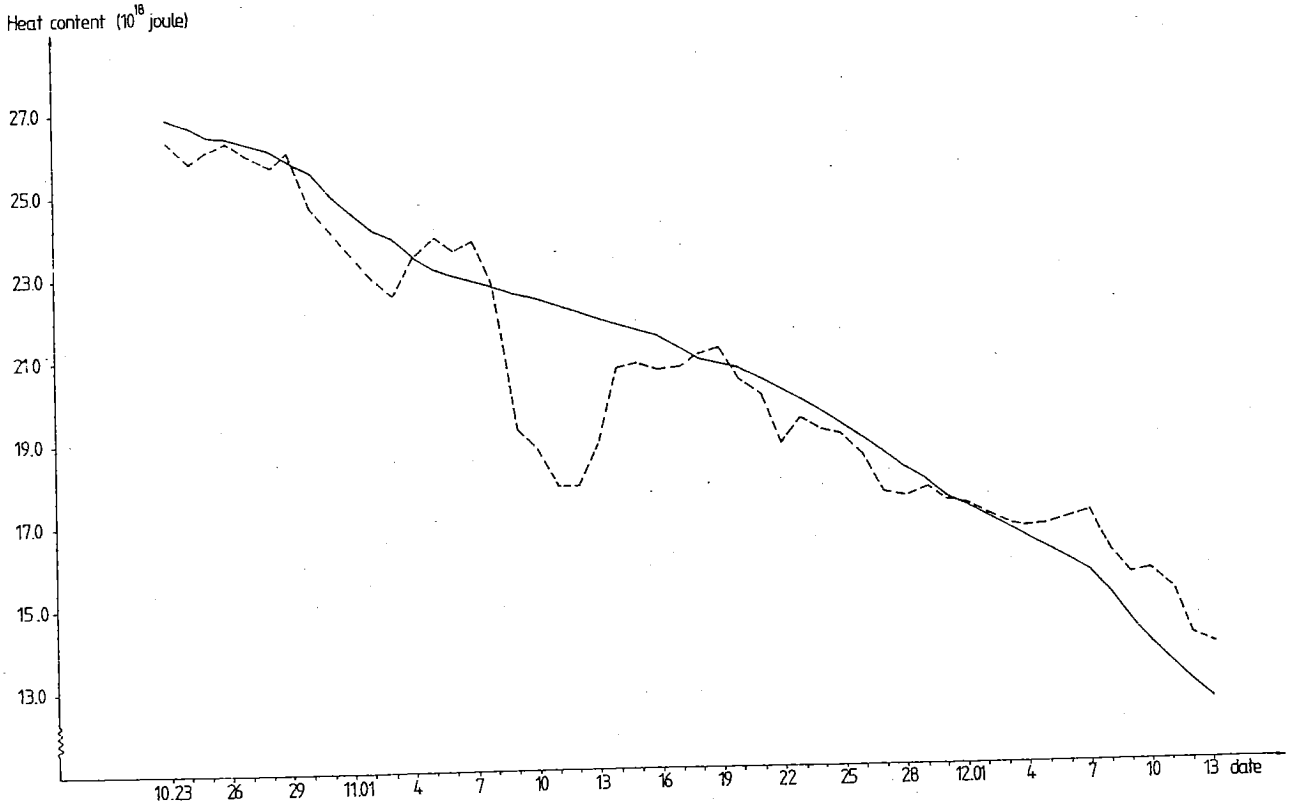

Figure 19 Heat content of the water column as given by the thermistor chain (dashed line) and expected variations due to the one-dimensional model (solid line).

6. SUMMARY AND CONCLUSIONS

The objective of this study was to examine cooling of the Bay of Bothnia during autumn. For this purpose temperature profiles were measured in the Bay of Bothnia during 52 days, from October 23 to December 13, 1979. The measured profiles have been compared to calculated ones, obtained by a mathematical model. The mathematical model is based on the conservation equations for momentum, heat and salt in their one-dimensional form and with an equation of state which is linear with respect to salinity and quadratic with respect to temperature. Turbulent exchange coefficients are calculated with a kinetic energy-dissipation model of turbulence.

It was demonstrated by the model that salinity influences the cooling by lowering the temperature for maximum density and that a stable salinity gradient reduces the deepening rate of the wind

mixed layer and inhibits convection down to the bottom. This salinity effects are of great importance when modelling cooling in the Bay of Bothnia.

The measured temperature profiles clearly demonstrate the importance of both temperature and salinity gradients in the mixed layer dynamics during autumn cooling.

Most important factors for the cooling rate are the heat loss at the air-sea interface and the mixing depth. The mixing depth depends mainly on wind mixing, convection and restratification when cooling has passed the temperature of maximum density. All these processes could well be described by the mathematical model and the computed and observed temperatures therefore agreed well.

During five days, advective effects were found to have a strong influence on the measured temperatures. Such events show the weakness of one-dimensional models, which of course cannot handle advection.

To test the model further temperature data have been collected at two locations in the Sea of Bothnia during 1981/82. If the results from these measurements are successful and model calculations are in as good agreement with measurements as above, an operational model for calculating cooling of sea water in the Bay and Sea of Bothnia is planned.

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Nomenclature

Symbol	Meaning
a_i	constants
A_w	absorption function
b_i	constants
C	a constant
$C_{1\varepsilon}$, $C_{2\varepsilon}$, $C_{3\varepsilon}$, C_μ	constants in turbulence model
C_p, C_p^a	specific heat
C_C, C_E	heat transfer coefficients
C_d	drag coefficient
d	a constant
e_a, e_s	vapour pressure
f	coriolis parameter
g	gravitational acceleration
h	a constant
k	turbulent kinetic energy
l	length scale
L	latent heat of evaporation
N_i	amount of clouds of category i
N	total cloud coverage
Q	net heat flux
Q_s	net short wave radiation
Q_L, $Q_{L\uparrow}$, $Q_{L\downarrow}$	long wave radiation
Q_C	sensible heat flux
Q_E	latent heat flux
Q_O	solar constant
Q_s, Q_a	water vapour density
S	salinity
T	water temperature

Symbol	Meaning
T_a	air temperature
T_f	freezing point temperature
T_u	air turbidity
T_i	cloud function
T_R	transmission function
T_s	surface water temperature
$T_{\rho m}$	temperature for maximum density
t	time
U	mean wind velocity in x-direction
V	mean wind velocity in y-direction
$\lvert V^a \rvert$	mean wind velocity
$\overline{wu}$, $\overline{wv}$	Reynold stresses
$\overline{w\theta}$, $\overline{ws}$	turbulent transport of heat and salt
Z	vertical coordinate
Z'	zenith angle
α	a constant
α'	sea surface albedo
β	a constant
ε	rate of dissipation of k
ε_m	water emissivity
κ	molecular thermal diffusivity
κ_s	molecular diffusivity of salt
μ_{eff}	effective dynamical eddy viscosity ($\mu_{eff}=\rho\nu_T$)
ν	molecular kinematic viscosity
ν_T	turbulent kinematic eddy viscosity
ρ_a	air density
ρ, ρ_o	water density
σ	Stefan Bolzmans constant

Symbol	Meaning
σ_k, σ_ε, σ_T, σ_S	Prandtl/Schmitt numbers
σ_t	water density (ρ-1000)
τ_x, τ_y, π	wind stresses

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