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DYNAMIC LOADS AND RESPONSE OF ICEBREAKER SISU DURING CONTINUOUS ICEBREAKING

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F O R E W O R D

A problem often encountered by ships when breaking ice, is violent vibrations. This problem is the subject of research report No 37 of the Winter Navigation Research Board, which herewith is presented.

As a step towards a method for predicting these vibrations at the design stage of a ship, the vibration modes and dynamic iceloads on the icebreaker SISU when operating in ice, are analysed.

The Winter Navigation Research Board expresses its thanks to the author and his colleagues at the Technical Research Centre of Finland.

Helsinki and Norrköping, October 1982.

Jan-Erik Jansson   Kaj Janérus

LIST OF CONTENTS

LIST OF SYMBOLS

$[C]$	damping matrix
F_o	force delivered by a shaker
$f = \frac{\omega}{2\pi}$	frequency
g	damping ratio
H	transfer function
$[K]$	stiffness matrix
$[K_I]$	damping matrix defined as imaginary part of the complex stiffness matrix
i,k,j	indexes
i	$= \sqrt{-1}$
m	generalized mass
$[M]$	mass matrix
N	number of natural modes
n	number of degrees of freedom
Q	generalized force
$\{q\}$	vector of external forces
S	spectral density
t	time
u	generalized coordinate
x	displacement
y	coordinate measured along the ship length
β	frequency ratio
$\{\phi\}$	natural mode vector
ω	$= 2\pi f$ frequency
ξ	damping ratio

1. INTRODUCTION

Ice-going ships are often subjected to heavy, multifrequency vibrations. These vibrations apart being annoying for the crew may also cause local structural damages or a failure of the instrumentation. For a slender ship these vibrations can be moreover associated with a considerable stress level in the deck plating in the midship area.

In order to be able to predict the vibration and vibratory stress level experienced by ship operating in ice, at an early stage of her design, a knowledge of dynamic ice loads and other relevant parameters, such as for instance damping, is needed.

The main objectives of this study were as follows

- to identify the vibration modes and to evaluate the vibration level of the icebreaker SISU operating in a level ice of the Gulf of Bothnia

- to develop a method aimed for an evaluation of the dynamic ice loads causing ship operating in ice to vibrate.

2. THEORETICAL BACKGROUND

The equations of motion of a linear multi-degree-of-freedom (MDOF) system subjected to external excitation forces $\{q(t)\}$ can be written as follows

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{q(t)\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are n by n mass, damping and stiffness matrices

and $\{x\}$, $\{\dot{x}\}$ and $\{\ddot{x}\}$ are displacment, velocity and acceleration vectors comprising n elements each.

Applying the mode superpostion method /1,2/ matrix Eq. (1) is reduced to a set of N uncoupled second order equations of motion

$$\ddot{u}_j+2\xi_j\omega_j\dot{u}_j+\omega_j^2 u_j = \frac{1}{m_j} Q_j(t) \quad ; \; j= 1,2...N \tag{2}$$

where u_j is jth generalized cooridnate (contribution of jthnatural mode of the MDOF system in the dynamic response)

ξ_j is relative damping coefficient associated with jth mode

ω_j is jth natural frequency of the system

$Q_j(t) = \{\phi_j\}^T\{q(t)\}$ jth generalized force

$\{\phi_j\}$ is jth natural mode of the system

m_j is the jth generalized mass

When the load vector $\{q(t)\}$ is known and given explicitly as a function of time then a set of Eqs (2) can be solved one by one. Thus the response in terms of displacement at an arbitrary point M is given by the series

$$x(M;t) = \sum_{j=1}^{N} u_j(t)\cdot\phi_j(M) \tag{3}$$

An experience of the ship vibration measurements proves that ship vibratory behaviour can be in the majority of cases regarded as that of a linear MDOF system. Thus a prediction of ship vibration can be accomplished if ship dynamic properties (generalized parameters) and dynamic loads are known. The latter can be of three different types:

a) Periodic loading generated by propeller and machinery is in most cases the cause of vibration problems aboard the ships. Harmonic load $Q_j(t) = Q_j \exp(i\omega t)$ yields a harmonic solution of Eqs. (2) and thus results in a steady harmonic vibration of ship.

b) Slamming, blast or ship-ice impact, apart local structural damage, may generate transient, multifrequency vibration of ship. In this case vibratory response can be calculated by a step-wise integration of Eqs. (2) in time domain.

c) Ship advancing through level ice is subjected to a continuous train of ice impacts, acting primarily in the bow area, and causing multifrequency vibration of the ship hull /3/.

2.1 Ice-induced vibration as a stationary random process

Although the breaking pattern of uniform level ice in the vicinity of ship bow exhibits a certain regularity, the forces having an origin in ice-hull interaction are very difficult to evaluate. The vibratory response comprising few first symmetrical natural modes of ship indicate that ice loads can be regarded as a random broadband process. If the ship speed and ice conditions do not change this process can be moreover regarded as a stationary one.

2.2 Response of a ship subjected to stationary random ice loads

We shall designate by $S_f(y,y';\omega)$ the spectral density of the vertical component of a dynamic load acting at points y and y' of ship advancing in level ice. Using the generalized force concept power spectral densities of generalized load are obtained /6/.

$$S_{Q_jQ_k}(\omega) = \int_{l'}\int_{l'} S_f(y,y';\omega)\phi_j(y')\phi_k(y)\,dy'\,dy \qquad (4)$$

where the integration is performed over the hull length l' affected by the ice load.

For a discretized structure and load formula (4) can be written in a matrix form

$$S_{Q_jQ_k}(\omega) = \{\phi_j\}^T\,[S_f(\omega)]\,\{\phi_K\} \qquad (4a)$$

where $[S_f(\omega)]$ is a matrix built up by the spectral densities of the M point forces into which the external load is distributed.

The response of a MDOF system in terms of the generalized coordinates u_j is given by

$$S_{u_ju_k}(\omega) = \frac{S_{Q_jQ_k}(\omega)}{m_jm_k\cdot(\omega_j^2-\omega^2-2i\xi_j\omega_j\omega)(\omega_k^2-\omega^2+2i\xi_k\omega_k\omega)} \qquad (5)$$

where $S_{u_ju_k}(\omega)$ are the power spectral densities of the generalized coordinates or in other words power spectral densities of the modal contributions in the response.

Denominator in the formula (6) can be understood as an inverse of a single element of the N-dimensional transfer matrix $[H(\omega)]$ of the MDOF system.

The power spectral density for response x(t) is obtained as /6/

$$S_x(y,y';\omega) = \sum_j^N \sum_k^N S_{u_j u_k}(\omega)\cdot\phi_j(y)\cdot\phi_k(y') \qquad (6)$$

where double summation is done over the number of modes N involved in the response. It is interesting to note that the transfer function $[H(\omega)]$ is a Hermitian matrix and

$$S_{Q_j Q_k}(\omega) = S^*_{Q_k Q_j}(\omega)$$
$$S_x(y,y';\omega) = S^*_x(y',y\ ;\omega)$$
$$S_{u_j u_k}(\omega) = S^*_{u_k u_j}(\omega)$$

where symbol * denotes a complex conjugate.

Root-mean-square value of the ship vibratory response at an arbitrary point M within the frequency bandwidth $\Delta\omega = \omega_2 - \omega_1$ can be obtained by an integration of (6)

$$x_{rms}(M) = \sqrt{\int_{\omega_1}^{\omega_2} S_x(M;\omega)\,d\omega}$$

2.3 Evaluation of the stochastic ice loads from the dynamic response of a ship

Spectral density $S_f(y,y';\omega)$ of the vertical component of dynamic load acting on a ship advancing in level ice can be evaluated following the procedure outlined above backwards. Let us assume that we know the transfer function of the ship composed of N sets of ship's generalized parameters corresponding to her N first vertical natural modes. These generalized parameters can be computed (applying the finite element method or equivalent) or evaluated experimentally by conducting a dynamic test of ship. The dynamic vertical response caused by stochastic loads of an ice field should be measured simultaneously in at least N points. These points should be selected so as not to coincide with any of the nodal points of N first natural modes. Computing N(N+1)/2 cross spectral densities of the response yields left side of Eq. (6). Equation (6) can now be rewritten in a matrix form

$$\{S_x(\omega)\} = [A]\{S_{uu}(\omega)\} \tag{6a}$$

$$\{S_{uu}(\omega)\} = [A]^{-1}\{S_x(\omega)\} \tag{6b}$$

Square matrix $[A]^{-1}$ of the dimension N^2 is an inverse of a real $[A]$ matrix independent of frequency and built up by the products of modal amplitudes according to formula (6). $\{S_x(\omega)\}$ is a vector of dimension N^2 composed of measured spectral densities of response and its complex conjugates. The unknown vector of spectral densities of the generalized coordinates $\{S_{uu}(\omega)\}$ can be evaluated now, for each frequency ω, applying Eq. (6b).

Power spectral densities of the generalized forces producing the measured response $S_x(\omega)$ are then given by

$$S_{Q_jQ_k}(\omega) = S_{u_ju_k}(\omega)\cdot m_j m_k\cdot(\omega_j^2-\omega^2-2i\xi_j\omega_j\omega)\cdot (\omega_k^2-\omega^2+2i\xi_k\omega_k\omega) \tag{7}$$

A transfer of ice load from the domain of generalized forces into the M point forces, given by a Hermitian matrix $S_f(\omega)$ of dimension M, can be obtained making use of formula (4a). The procedure is similar to the one already applied in evaluating the unknown vector of spectral densities of generalized coordinates (formula 6b). When the number of sought point forces equals the number of natural modes involved in the response i.e. M = N, then a solution can in principle be obtained as a result of solving two sets of N(N+1)/2 linear algebraic equations. Real frequency independent coefficients of this system equal the products of modal amplitudes at the points of point forces application.

Repeating this procedure for the entire investigated frequency bandwidth, with a frequency step equal to the resolution of Fast Fourier Transform (FFT) performed on the vibratory hull response, yields spectral densities of point forces at a number of discrete frequencies.

Root-mean-square value of the point force acting on ship at point y, within the frequency bandwidth $\Delta\omega = \omega_2 - \omega_1$, can be obtained by an integration

$$f_{rms}(y) = \sqrt{\int_{\omega_1}^{\omega_2} S_f \cdot (y;\omega)\, d\omega} \tag{8}$$

The procedure presented above is unfortunately very sensitive to the modal parameters of the ship, to the location of the measuring points and to the accuracy of the Fast Fourier Transform of the measured response. It can be, however, simplified and thus made less sensitive to these factors assuming a location of the "bow force" equivalent to a distributed over the ship length 1' ice loads. Spectral density of this single force applied at point y_f can be evaluated with a formula

$$S_f(y_f;\omega) = \frac{S_{Q_j Q_k}(\omega)}{\phi_j(y_f) \cdot \phi_k(y_f)} \tag{9}$$

For the ith natural frequency and close vicinity of it $j = k \equiv i$ can be taken. For the frequency ω located between two natural frequencies ($\omega_i < \omega < \omega_{i+1}$) the corresponding cross-spectral density should be applied i.e.

$$S_f(y_f;\omega) = \frac{S_{Q_i Q_{i+1}}}{\phi_i(y_f) \cdot \phi_{i+1}(y_f)} \tag{9a}$$

3. EXPERIMENTAL EVALUATION OF THE GENERALIZED PARAMETERS

Dynamic properties of icebreaker SISU were identified by a shaker test conducted in an open, deep water on the 21st and 22nd of November, 1980. Within the frequency range from 2 to 10 Hz the first four natural frequencies associated with the symmetrical vertical modes were encountered. The modes were measured in 50 points located so as to represent the vibratory behaviour of the entire ship.

3.1 Main particulars of the ship and test condition

Length overall	104.6	m
Length, DWL	96.0	m
Breadth, Max.	23.8	m
Breadth, DWL	22.5	m
Draft, Max.	8.3	m
Draft, DWL	7.3	
Propeller shaft output	16.2	MW
Speed in open water	18.7	knots

Draft of the ship during test	7.5 m (even keel)
Depth of water	45 m

3.2 Vibration excitation and measuring system

A scheme of measuring system for ship dynamic testing, developed at the Ship Laboratory of Technical Research Centre of Finland, is shown in Fig. 2. It comprises three sections.

3.2.1 Vibration excitation section

Vibration is excited by a single hydraulic type shaker (see Fig. 3) welded firmly to the stiff structural members of ship.

Excitation force delivered by the shaker is generated by a vertically vibrating mass which follows a signal generated by a desk-top computer controlled function synthesizer. The motion of the mass is produced by a hydraulic cylinder and measured by a motion transducer of Hottinger. The shaker is presented in Fig. 3 and a typical plot of a force delivered by it vs. frequency is shown in Fig. 4. The shaker is remotely controlled and can deliver sinusoidal or random (white-noise type) excitation. The latter type of excitation is very convenient to use when a fast identification of the first natural frequencies of ship is needed. Sinusoidal type of force, with the entire power of the shaker concentrated on one frequency, is, however, far more efficient for a dynamic testing of ships.

3.2.2 Measuring section

Vibration is measured by five seismic type, that is measuring displacements, vibration transducers (B3 model pick-ups of Hottinger). Each of them was individually calibrated. Calibration charts of them in a form of analytical expressions fitting the measured data are stored as a subprogram in a data analyzing program. The signals of the vibration pick-ups and the motion transducer are amplified by a six channel KWS 6E5 amplifier of Hottinger.

3.2.3 Data processing and recording section

Signals of all the transducers (5 vibration pick-ups and motion transducer) are read and processed on-line by a 3052 Automatic Data Acquisition system of Hewlett-Packard comprising desk-top computer, system voltmeter, scanner and digital plotter-printer. The on-line analysis includes Fourier analysis and calibration of the vibration and motion signals. Processed data are stored on the cassette memory of the computer. Moreover all signals are saved in analog form on a tape by an instrument tape recorder HP3968A.

Vibration spectra computed by a HP3582A dual channel spectrum analyzer are observed during the measurements.

3.3 Measuring procedure

The exciter was located at the centre line of the 2nd deck, at frame No. 118.

Each of the five vibration pick-ups was assigned a group of ten measuring points. Vibration was measured, simultaneously by all transducers, in 10 steps. Location of the measuring points is shown in Fig. 1.

In the first two steps resonant frequencies were identified by sweeping the frequency range from 2 to 10 Hz with a step of 0.03 Hz. For the remaining measuring steps the vicinity of each natural frequency was swept slowly increasing the frequency of shaker by very small steps.

3.4 On-line analysis of the measurements

Analysis of all vibration and motion transducers signals was conducted on-line as described in section 3.2.3. Results of the analysis in a form of a file corresponding to each measuring step containing
- frequencies of the shaker
- force amplitudes and phases related to the driving signal
- amplitudes and phases of the five vibration transducers

were stored on cassette memory of the desk-top computer.

Computer program executed immediately after the measurements processed the preanalyzed and stored on cassettes data. This final on-line analysis included:

- calculation and plotting of vibration amplitude per unit force of shaker excitation spectra
- calculation of vibration phase related to the excitation force and plotting the vibratory response of ship at the selected measuring points in a polar form.

3.5 Modal analysis

The main task of the investigation, i.e. modal analysis of icebreaker SISU, was conducted on the data stored in digital form during the shaker test. Modal parameters that is natural frequencies and modes, generalized dampings and masses were identified applying the method presented below.

The vibration of a ship, in terms of displacement at an arbitrary point M can be expressed as follows:

$$x(M,t) = \sum_{k}^{N} \frac{\{\phi_k\}^T\{F_o\}((1-\beta_k^2)-ig_k)}{m_k\omega_k^2((1-\beta_k^2)^2+g_k^2)}\phi_k(M) \quad e^{i\omega t}$$

$$= \sum_{k}^{N} (Re(x_k(M)+iIm(x_k(M)\cdot\phi_k(M) \quad e^{i\omega t} \tag{10}$$

where $\{F_o\}$ = vector force of shakers delivering the harmonic excitation of frequency ω or other single harmonic excitation,

m_k = $\{\phi_k\}^T[M]\{\phi_k\}$= kth generalized mass,

ω_k = kth natural frequency

β_k = ω/ω_k = frequency ratio

g_k = $\alpha\omega_k = \frac{1}{m_k\omega_k^2}\{\phi_k\}^T[K_I]\{\phi_k\}$, generalized damping

$g_k = 2\xi_k$ for $\beta_k = 1$

ω = $2\pi f$

$[K_I]$ = damping matrix being an imaginary part of a complex stiffness matrix

$\{\phi_k\}$ = kth natural mode of ship.

When conducting dynamic tests with a single shaker, it is convenient to plot vector response (displacement per unit force) loci in the complex plane. When the vibrator is sweeping through a certain natural frequency, the tip of the total response vector tends partly to describe a circle corresponding to this mode as the components due to other modes remain more or less constant.

The natural frequency can be found as a point on the response loci corresponding to the maximum increase of its length per unit frequency of excitation.

Drawing the diameter of a circle through the point of natural frequency and transferring the origin leads to a single degree of freedom system which represents the ship vibrating with a single mode only.

As an example, the vector response plot of 5000 dwt tanker is shown in Fig. 5. (reference /7/)

Having reduced the vibratory response to a single degree of freedom system, the series in equation (10) reduces to one term only, i.e.:

$$x'(M,t) = \frac{\{\phi_k\}^T\{F_o\}((1-\beta_k^2)-ig_k)\,\phi_k(M)}{m_k\omega_k^2((1-\beta_k^2)^2+g_k^2)}\,e^{i\omega t} \qquad (11)$$

$$= (Re(x')+iIm(x'))e^{i\omega t}$$

and for resonance $(\beta_k = 1)$

$$x'(M) = iIM(x').$$

If we normalize the natural mode in such a way that the modal displacement at a point M equals the measured displacement at the same point, which is

$$\phi_k(M) = |Im(x'(M)|_{\beta_k = 1}$$

then the kth generalized mass is given by the formula

$$m_k = \frac{F_o}{2\omega_k} \frac{dRex'(M)}{d\omega} \tag{12}$$

and the kth generalized damping by

$$g_k = \frac{\phi_k(M) \cdot F_o}{m_k \ \omega_k^2} \tag{14}$$

Having identified generalized parameters, we can use equation (10) inserting in it actual excitation forces $\{F_o\}$ to calculate the ship response.

The procedure described above requires the measurement of vibration amplitude and phase related to the excitation of the shaker. Moreover, the excitation force amplitude must be known in order to determine the generalized masses.

3.6 Results of the shaker test

Within the investigated frequency range from 2 to 10 Hz four natural frequencies were found. They are listed in Table 1. Examples of typical vibration amplitude spectra are presented in Figs 6 - 9. These plots exhibit the peaks at the same four frequencies listed in Table 1 indicating that the vibration modes associated with them are global (main girder) vibration shapes of ship.

Vibratory response of ship at the selected measuring points, in a form of polar plots, is presented in Figs 10 - 13. First four symmetrical modes of ship obtained from the polar plots drawn for each measuring point and each resonance are presented in Figs 14 - 17. Each mode is scaled so that the modal displacements equal the measured values. Modal scaling factors i.e. generalized masses and modal dampings are given in Table 1 and Figs 14 - 17. An example of determining the modal parameters for the first natural mode is shown in Fig. 18.

4. MEASUREMENTS OF ICE-INDUCED VIBRATION

Measurements of the vibratory response of SISU to ice-induced excitations were conducted during her normal operation in the Gulf of Bothnia from 15th to 19th of February, 1979 and from 27th to 29th of January, 1981.

4.1 First measurement

The main goal of the measurment was to gain the knowledge about the vibration level experienced by an icebreaker and to identify the vibration modes of her associated with the ship-ice interaction /3/.

During the measurement ship most of the time followed newly frozen old channel in level and compact ice. Ice thickness and speed of the ship were varying.

Vibration was measured by two transducers (B3 type pick-ups of Hottinger). One of them was located at point REF marked on Fig. 1 and not moved during the measurements serving as a reference signal generator. Second pick-up was measuring vibration in rest of the 28 points. Measuring points were located at the centre line of ship or as close to it as it was possible.

Amplified signals of the vibration pick-ups were recorded by an HP 3968A instrument tape recorder. In order to obtain a representative sample, the recording time for each of the measuring points was relatively long and equal to about 30 minutes.

Recorded vibration of the wandering and reference pick-ups was analyzed with a dual-channel spectrum analyzer HP 5420A.

As a result a set of transfer functions between the signals of these two transducers was obtained. These transfer functions i.e. vibration amplitudes ratios and relative phases versus frequency, proved to be practically independent of the ice conditions and ship speed.

An example of the computed transfer function is shown in Fig. 19.

A knowledge of the multi-dimensional transfer function and the spectrum of vibration amplitude for the reference point enabled to determine ship vibratory response associated with each distinct frequency. This procedure is outlined in Fig. 20 taken from reference /3/. An example of typical spectrum of vibration measured at the reference point is shown in Fig. 21.

The vibration modes of ship associated with the frequencies corresponding to four distinct peaks of the amplitude spectra are presented in Figs 22 - 25.

First two vibratory profiles, given in Figs 22 and 23, coincide well with two lowest natural modes of ship. Their resonant (natural) frequencies are f_1 = 2.75 Hz and f_2 = 5.10 Hz respectively. Hull vibration causes the rigid-body vibration of superstructure and funnel in the longitudinal direction (in-phase for two-node and out-of-phase for three-node vibration).

Four node bending mode of hull is shown in Fig. 24. It occurs at frequency f_3 = 6.80 Hz. The response of the ship to ice-impacts is more complex now. Vibration profile does not coincide with the natural mode because of phase differences of the response measured at different points. That is why the vibratory behaviour of ship at this frequency is presented at three, equally spaced instants covering a half of the vibration period. Lowest block of superstructure, foundation of the funnel and the helicopter platform vibrate mainly vertically. Two uppest blocks of superstructure vibrate in longitudinal direction, too.

Last detected component of ship vibration due to ice-impacts is presented in Fig. 25. It is associated with the frequency f_4 = 8.44 Hz and corresponds roughly to five-node bending mode of hull. The amplitudes of hull vibration decrease with a distance from the bow increasing. Vibration of the superstructure follows qualitatively hull deflections. Angular deflections of two uppest blocks of the superstructure is differently oriented for each of them, at this frequency. Longitudinal vibration of the funnel is absent.

4.2 Second measurement

The main objective of this measurement was to obtain the necessary data for an evaluation of the stochastic ice loads according to the method presented at point 2.3.
Vibration was measured simultaneously by four B3 transducers located as shown in Table 2. The measuring points were selected so as not to coincide with any of the nodal points of the first four natural modes of ship. Power of the main engines was kept during the measurements constant and equal to about 80 % of the maximum power. Ship was advancing through the level ice and together frozen ice floes of the thickness from 30 to 50 cm. The contents of the ballast and fuel tanks was nearly the same as during the shaker test.

Two representative samples were selected for the analysis from the bulk of measured data obtained. The time length of each of them was about 20 minutes. Ship speed was 12 and 10 knots and ice thickness about 30 and 50 cm correspondingly.

Analysis of the measurements was conducted with a dual channel spectrum analyzer HP 5420A. Cross-spectral densities of the vibratory response were computed by it within a frequency bandwidth 0 - 16 Hz and with a resolution 0.0625 Hz. Analyzed spectral densities were transferred to a HP 9845B desktop computer via the HP interface bus and from it to the CDC Cyber 170 computer of the Technical Research Centre of Finland.

4.3 Dynamic ice loads derived from the measured ship response

The examples of the cross-spectral densitites of ship vibratory response are presented in Figs 26 - 31. They constitue six elements of the complex, sixteen elements vector $\{S_x(\omega)\}$ being the left side of Eq. (6a).

The unknown vector of spectral densities $\{S_{uu}(\omega)\}$ for four generalized cooridnates was evaluated according to Eq. (6b) applying the modal amplitudes obtained as a result of the shaker test. In other words complex vibratory response of ship was decomposed into the modal contributions.

Spectral density plots of the first generalized coordinate that is the contribution of the first flexural mode of ship in the vibratory response, for speeds 10 and 12 knots, are shown in Fig. 32.

As it can be clearly seen from these plots first natural frequency decreses in the presence of ice field. Moreover this decrease depends of the ice thickness. As the ship stiffness is independent of the vessels speed and ice thickness the conclusion can be drawn that the presence of ice increases the added mass vibrating with a ship hull. This added mass affects the natural frequencies and scaling factors of the natural modes, i.e. generalized masses. The latter can be evaluated as follows

$$f_s = \frac{1}{2\pi}\sqrt{\frac{k_g}{m_{gs}}} \quad ; \quad f_i = \frac{1}{2\pi}\sqrt{\frac{k_g}{m_{gi}}}$$

yields $$m_{gi} = m_{gs}\sqrt{f_s/f_i} \qquad (15)$$

where

f_s natural frequency of ship obtained from the shaker test

f_i natural frequency of ship in ice

k_g generalized stiffness

m_{gs} generalized mass of ship obtained from the shaker test

m_{gi} generalized mass of ship in ice.

Another interesting effect of the crushed ice field on the ship hull vibratory behaviour is exhibited by the plots of Fig. 32. We can assume that a spectral density of the ice loads, within the bandwidth covering the resonance peak, is nearly constant. This assumption is fully justified noting the fact that the amplitudes of a ship hull vibration in terms of velocity are an order of magnitude smaller than ship speed. That is hull vibration does not affect the ice loads. With this assumption the plot of spectral density of the generalized coordinate is equivalent to a square of transfer function of this coordinate /4, 6/. Using the half-power (bandwidth) method /1/ the value of generalized damping associated with the mode in question can be easily determined.

Generalized parameters of the icebreaker SISU in ice are summarized in Table 3. A noted affect of ice field on the damping values can be clearly seen. Ice thickness dependent values of the generalized dampings are few times higher than the structural damping values of the ship in open water.

Spectral densities of the ice-load equivalent vertical component of the bow force are presented in Fig. 33. The plots were obtained applying the four-dimensional transfer function of ship with the generalized parameters given in Table 3 and assuming that a single bow force is acting on point R1. Computed values of spectral density of the bow force are marked by dots and approximated by solid and dotted line for ship speed 12 and 10 knots respectively. A certain scatter in the computed values can be explained by an approximation of the ice loads by as single bow force and errors of the modal analysis. The bandwidth of these plots is restricted to the region relevant to the vibratory response of ship. The steepness of the obtained spectra in the low frequency region indicates that for the icebreaker SISU maximum ice loads act with the frequencies much below the first natural frequency. Spectral density of the bow force could be extended relatively easily to this low frequency region applying the same spectral approach. This could be accomplished by including the rigid body motion of ship elements in the MDOF transfer function and measuring the vibratory response of hull in two more points.

Root-mean-square values of the bow force for the frequency bandwidth from 2.75 to 7.5 Hz are 85 and 110 kN for ship speed 12 and 10 knots respectively. These values should be understood as the average values of load encountered by ship during the measurements. For lower ship speed and thicker ice the force is bigger but due to higher damping the vibratory response of ship is lower (refer to Fig. 32). The frequency content of the ice loads, as could be expected, depends on the ship speed. With the ship speed increasing the ice loads components corresponding to the higher modes of hull vibration increase.

5. CONCLUSIONS

The measurements of vibration conducted on the icebreaker SISU during her continuous icebreaking mode of operation showed that four lowest natural modes were excited by the ice loads. These ice loads can be with a good degree of accuracy approximated by an equivalent bow force fixed in space and having a continuous frequency content. This ice thickness dependent bow force has a maximum much below the first natural frequency of the icebreaker SISU. Thus only a small fraction of ice loads causes this ship to vibrate.

The presence of the breaked ice field is felt by ship structure moreover as a significantly increased modal damping ratios and a certain amount of added mass vibrating with the ship hull.

Dynamic loads derived in this study apply to the icebreakers of URHO class only. In order to be able to predict the vibratory response of the new designs the influence of ship power and bow geometry on ice loads should be studied.

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Table 1. Generalized parameters of icebreaker SISU in open water.

Mode number	Natural frequency Hz	Generalized damping g_j	Generalized mass $kg \cdot m^2$	Modal amplitude at the shaker location µm
1	2.916	0.0075	0.118	181
2	5.513	0.0121	0.02012	81
3	7.550	0.0111	0.00486	23
4	9.000	0.0108	0.0036	15

Table 2. Location of the measuring points during the 2nd ice-induced vibration measurements.

Transducer No.	Location
1	KK1V
2	KK1L
3	R
4	K7

Table 3. Generalized parameters of icebreaker SISU in ice.

Mode number	Ice thickness	Natural frequency Hz	Generalized damping g_j	Generalized mass $kg \cdot m^2$	Modal amplitude at point REF µm
1	30	2.760	0.053	0.121	181
	50	2.703	0.095	0.122	
2	30	5.150	0.054	0.0208	81
	50	5.078	0.112	0.0210	
3	30	7.067	0.046	0.00502	23
	50	6.594	0.084	0.00520	
4	30	8.700	0.044	0.00366	15
	50	8.312	0.076	0.00375	

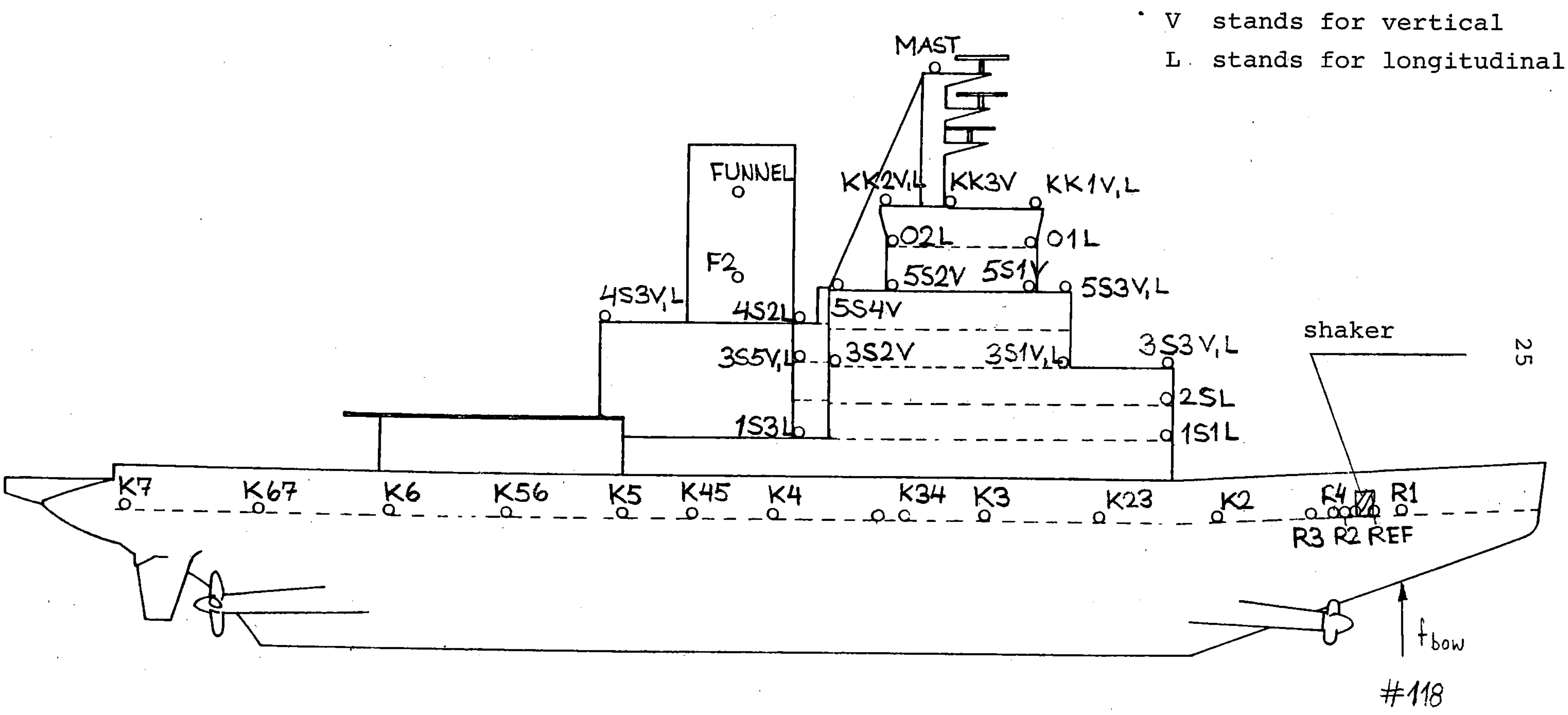


Fig. 1. Location of the measuring points.

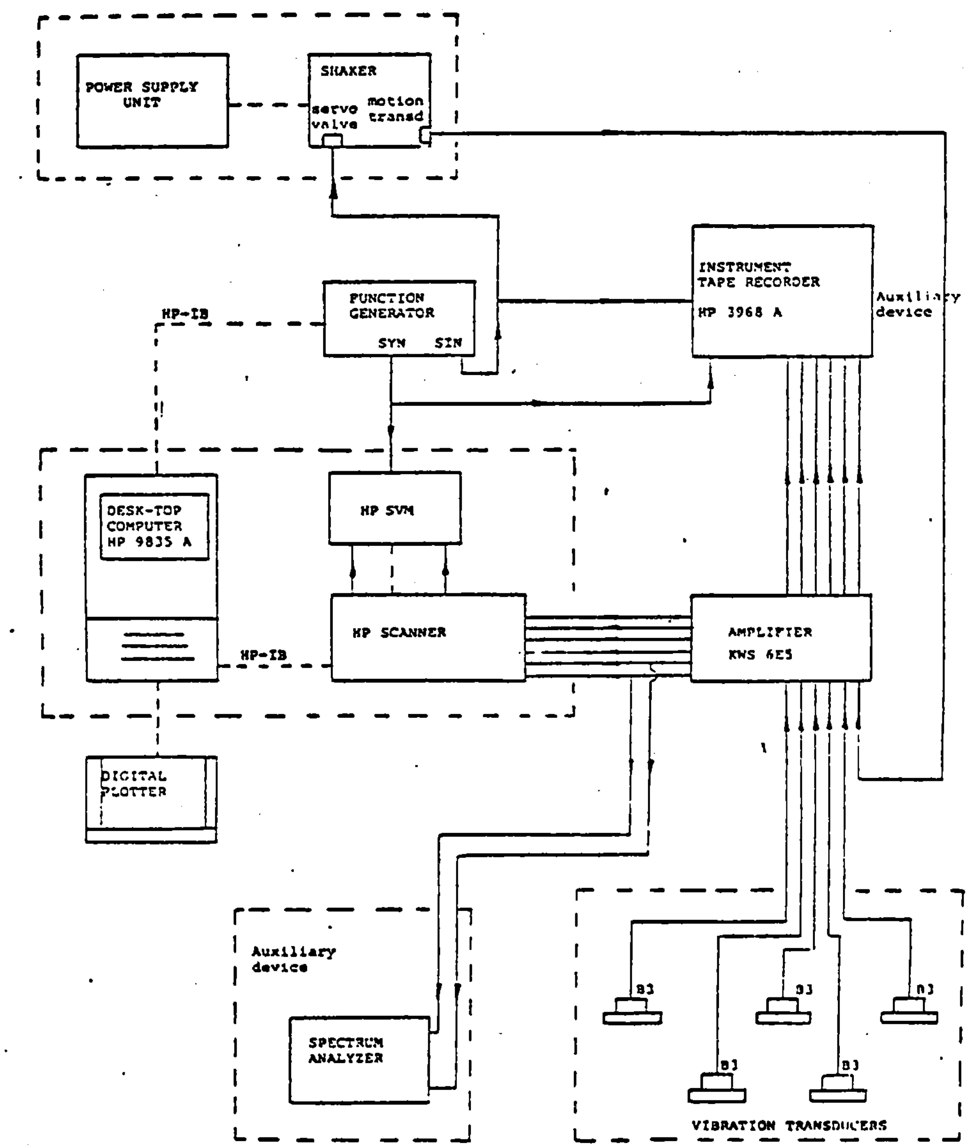


Fig. 2. Block diagram of the measuring system for shaker test.

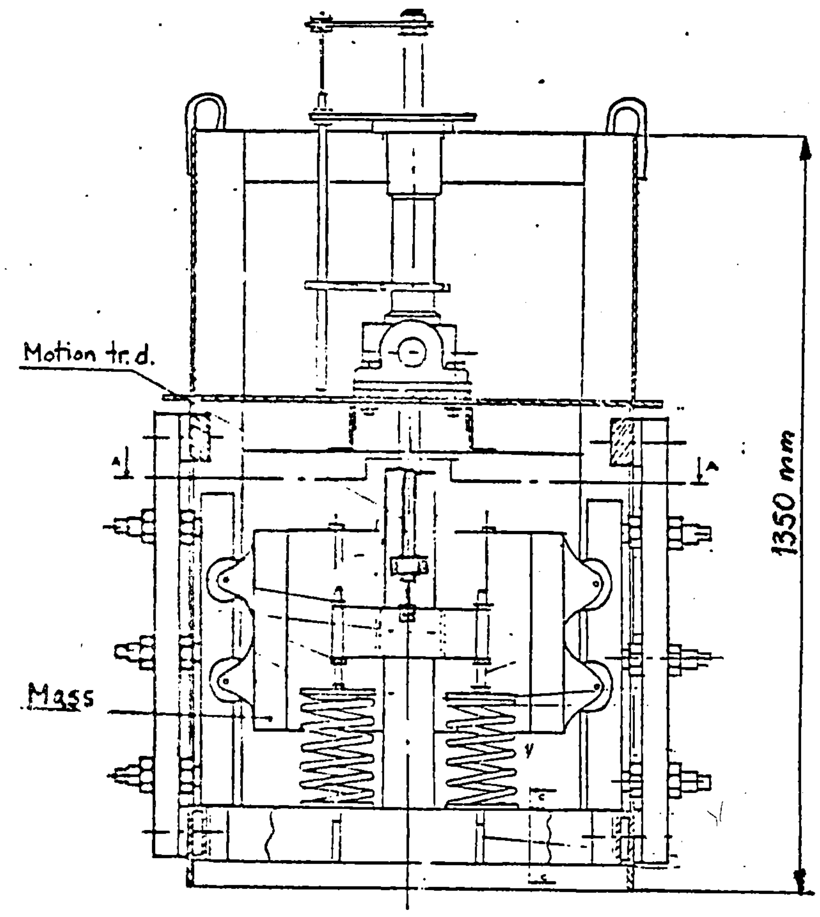


Fig. 3. Shaker for resonant testing of ships.

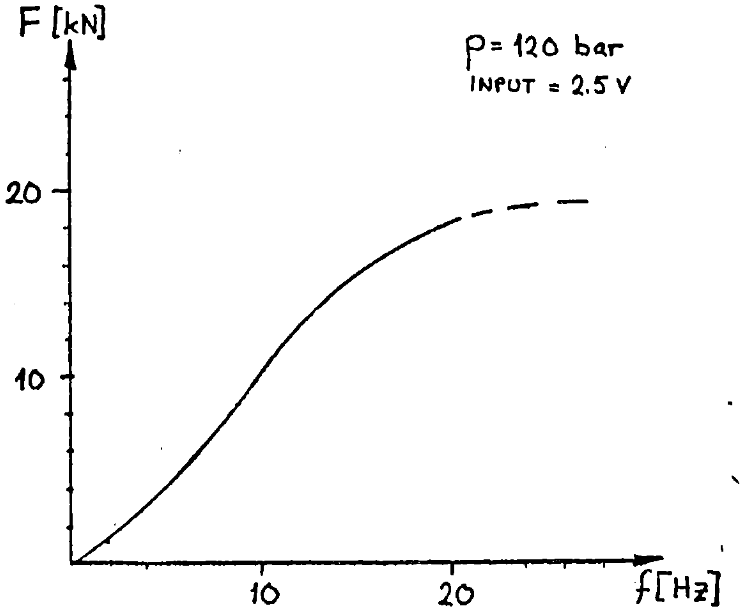


Fig. 4. Typical plot of force delivered by shaker.

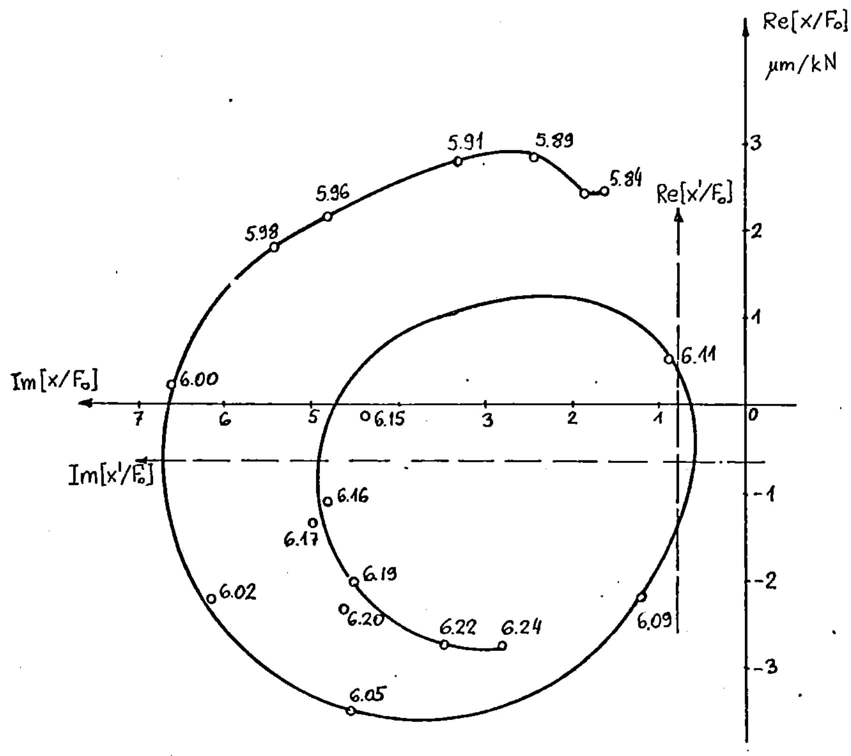


Fig. 5. Vibratory response of a 5000 DWT supply tanker.

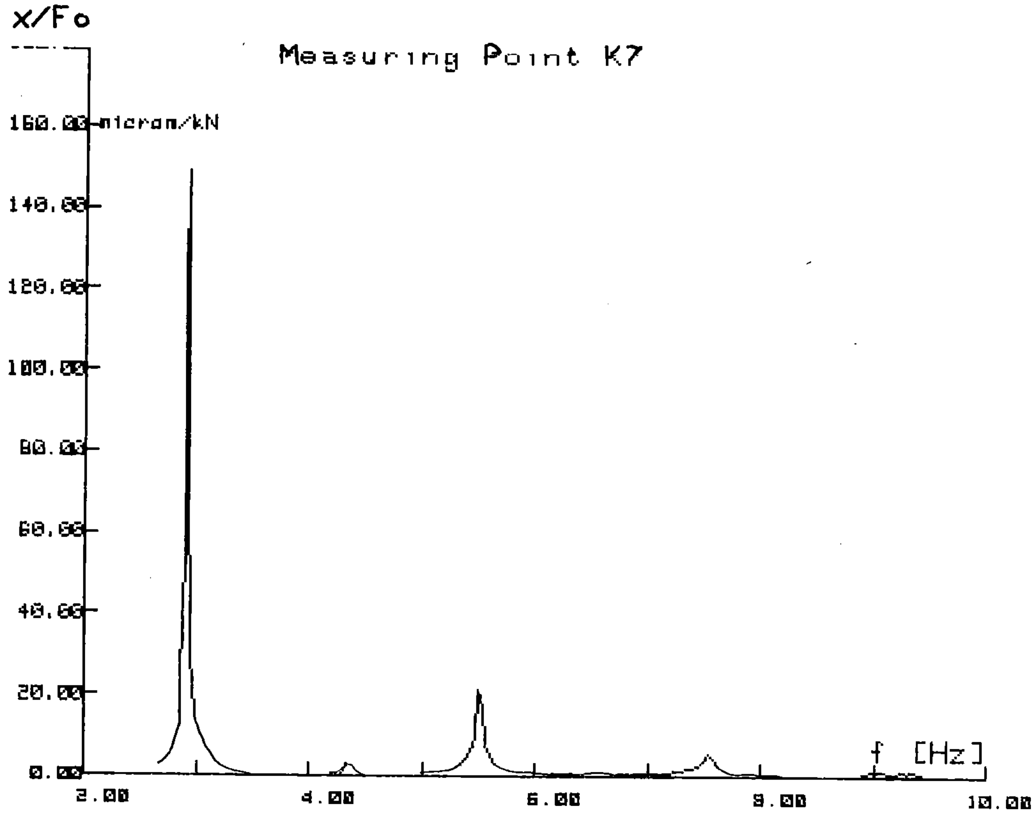


Fig. 6. Vibration amplitude spectrum of the response measured at point K7.

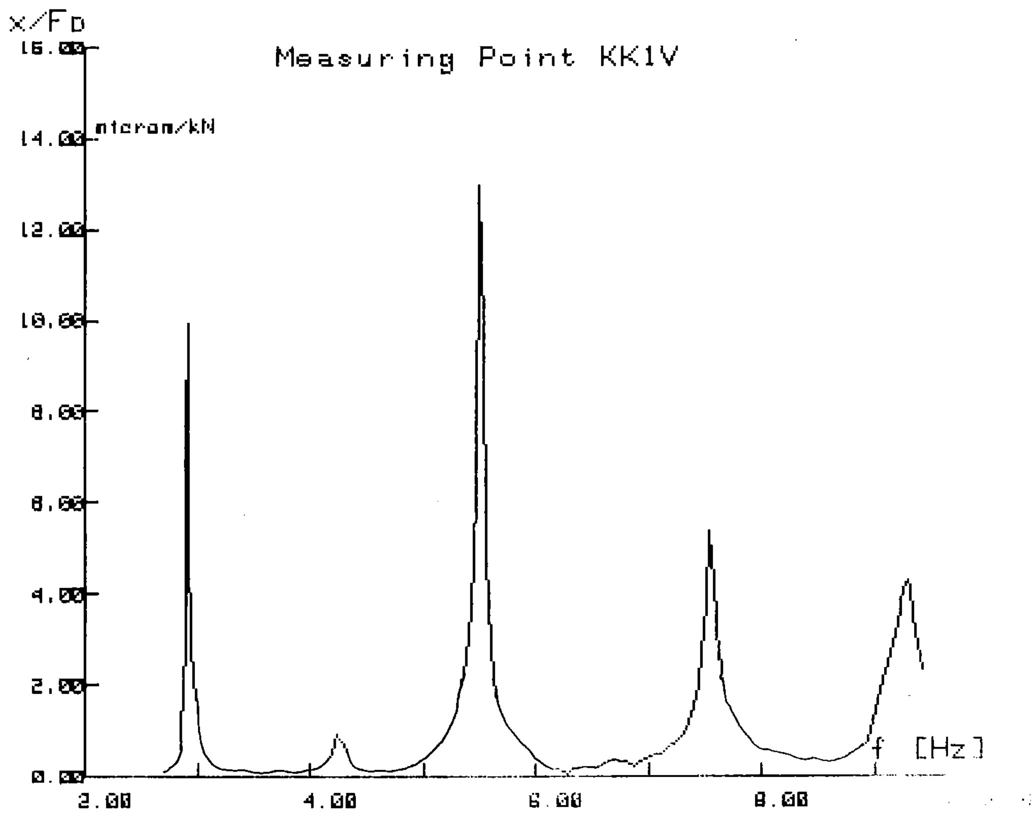


Fig. 7. Vibration amplitude spectrum of the response measured at point KK1V.

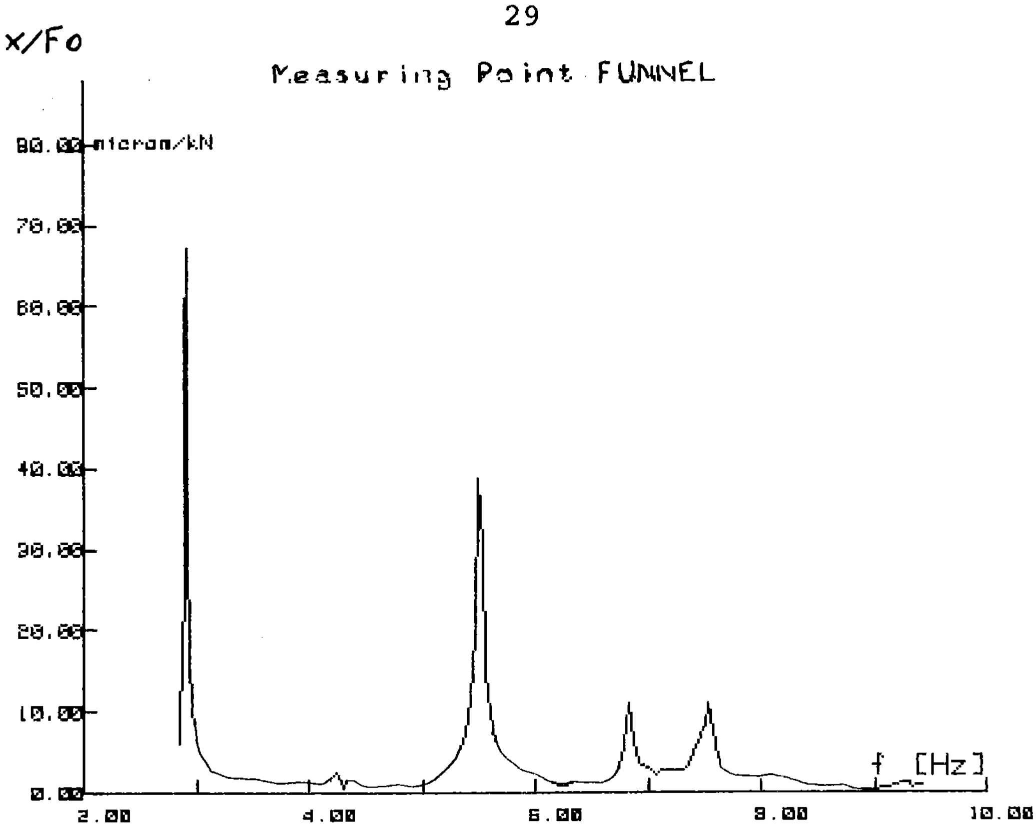


Fig. 8. Vibration amplitude spectrum of the response measured at point Funnel.

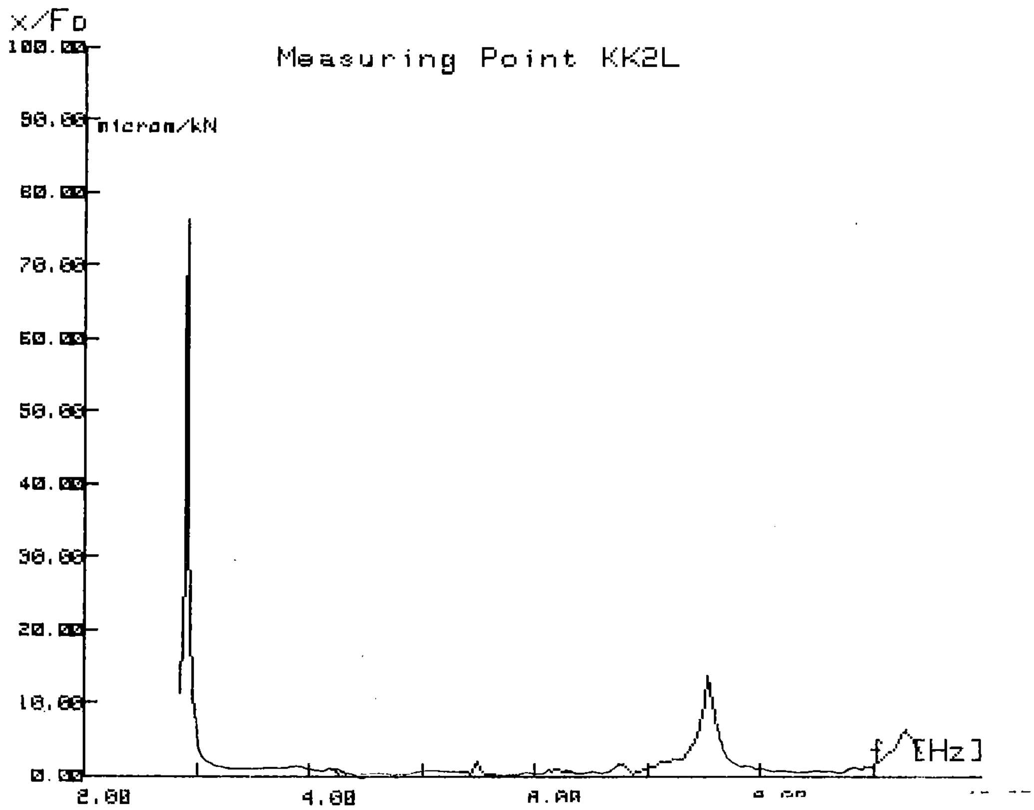


Fig. 9. Vibration amplitude spectrum of the response measured at point KK2L.

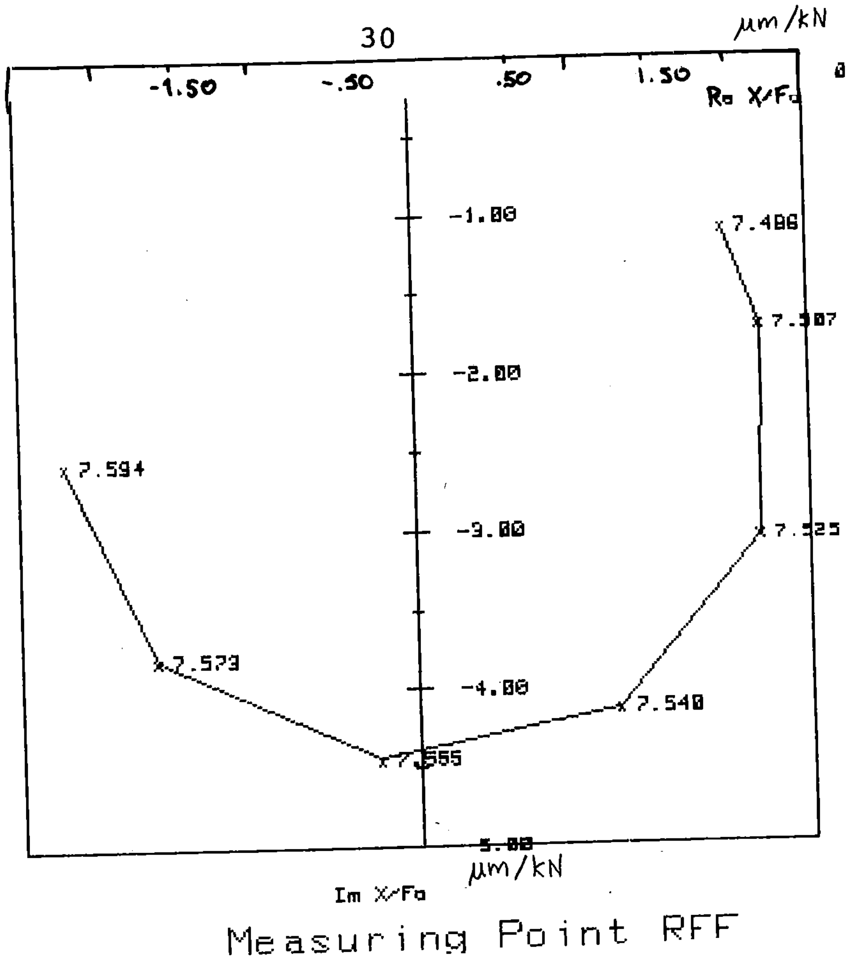


Fig. 10. Polar plot of the response measured at point REF at 3rd natural frequency.

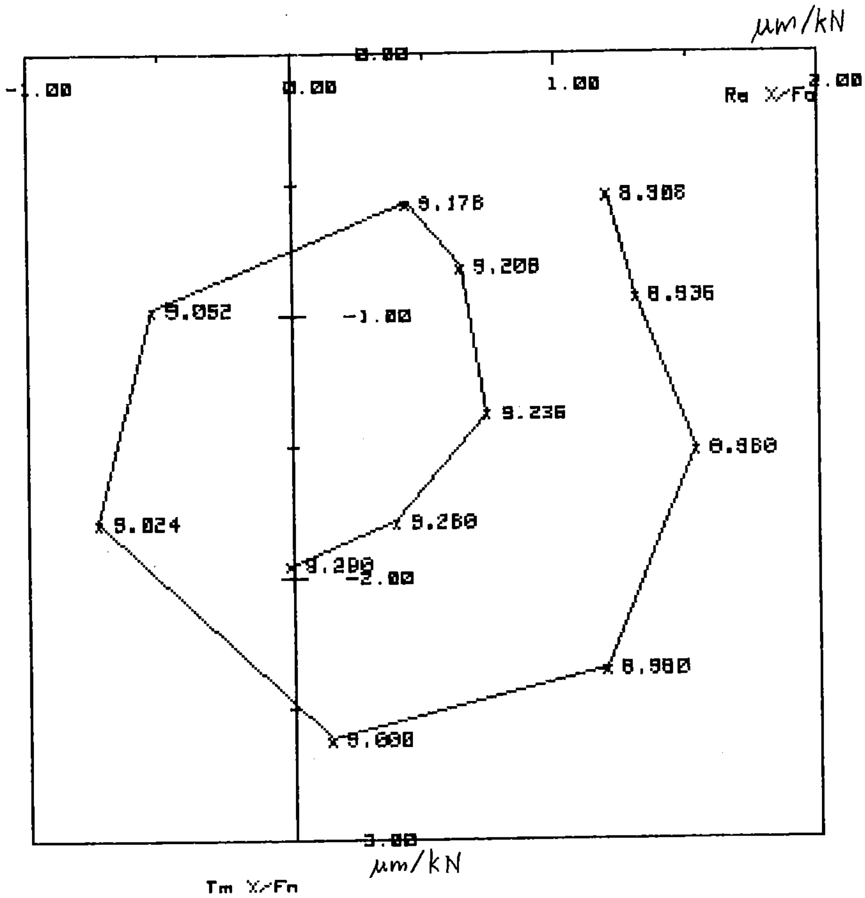


Fig. 11. Polar plot of the response measured at point REF at 4th natural frequency.

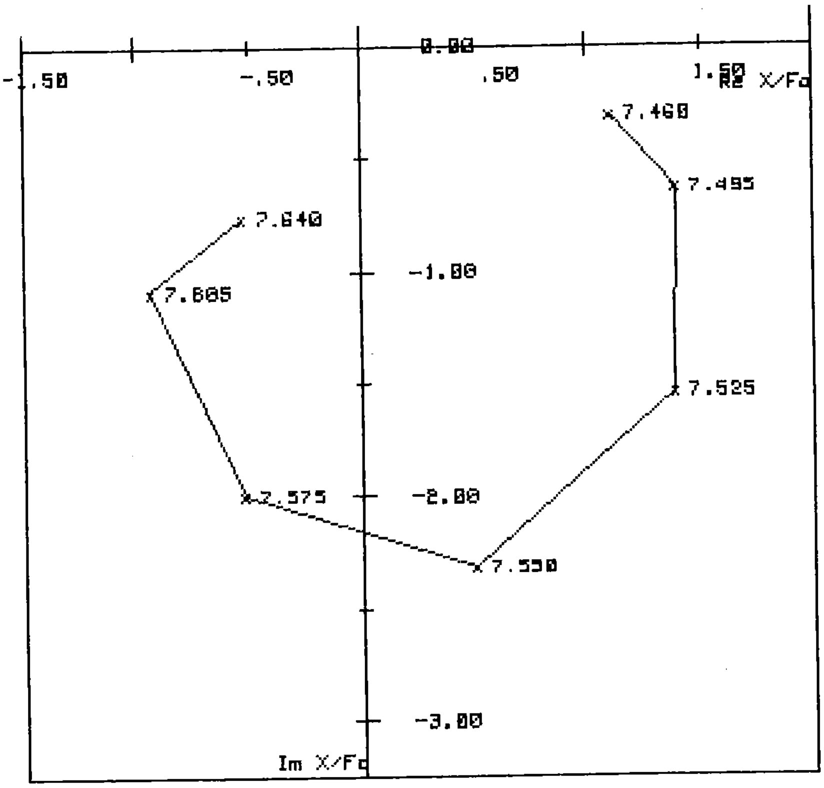


Fig. 12. Polar plot of the response at point 3S2V at 3rd natural frequency.

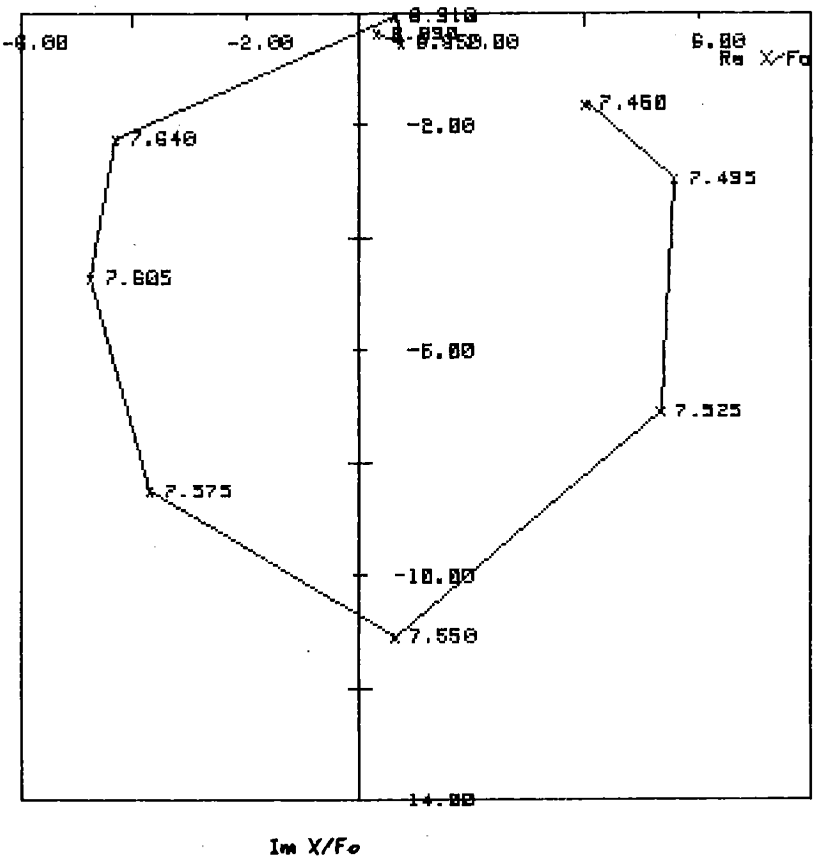


Fig. 13. Polar plot of the response at point KK1L at 3rd natural frequency.

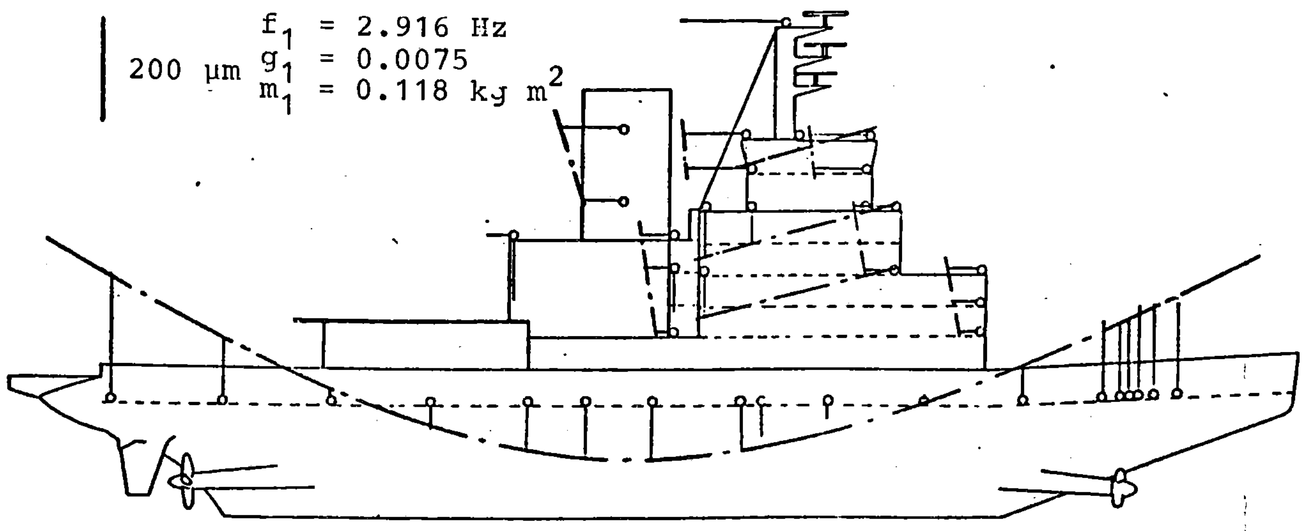


Fig. 14. First natural mode of icebreaker SISU.

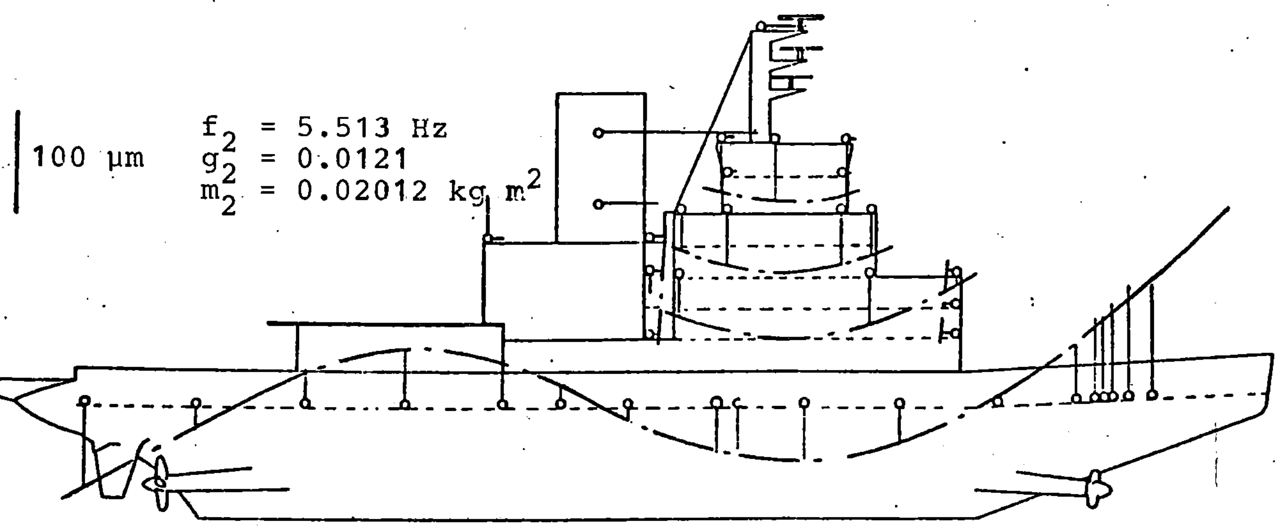


Fig. 15. Second natural mode of icebreaker SISU.

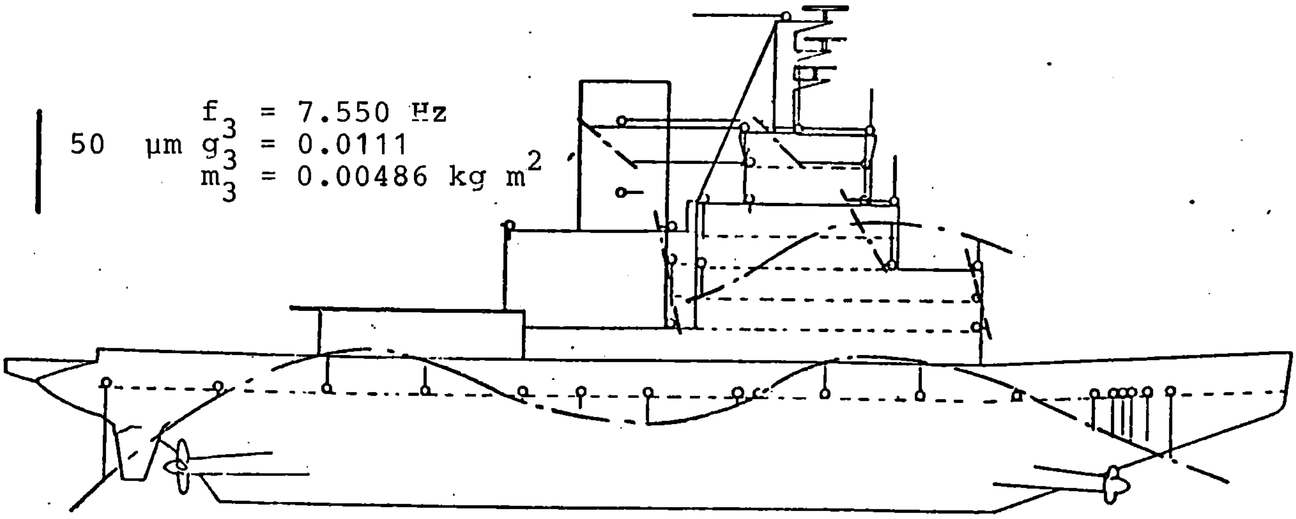


Fig. 16. Third natural mode of icebreaker SISU.

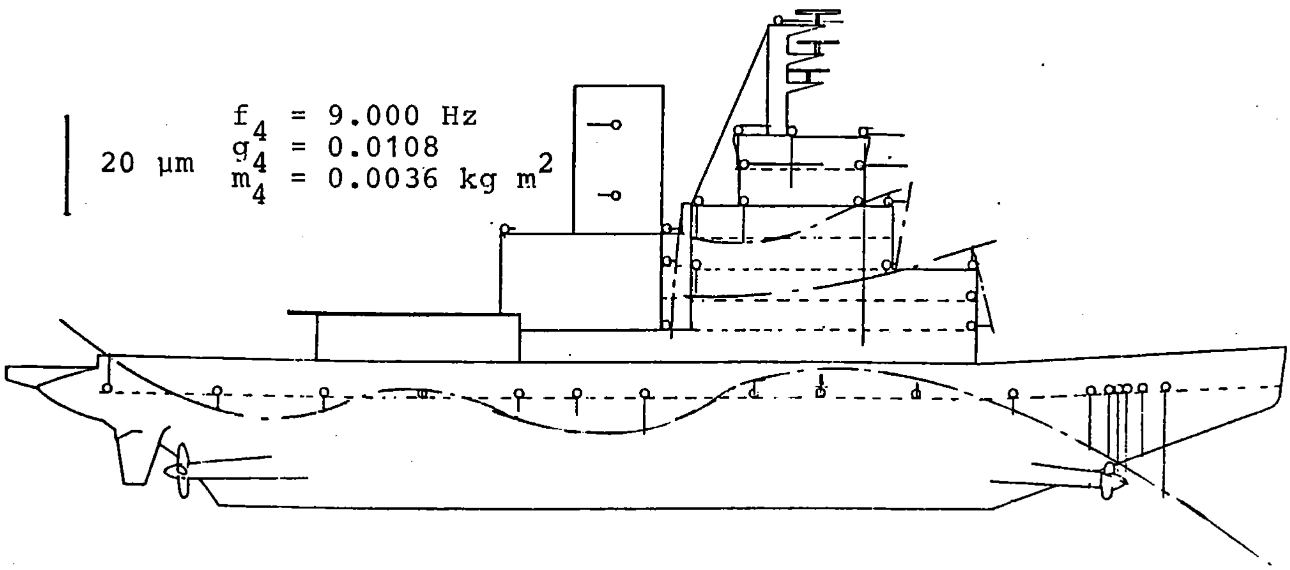
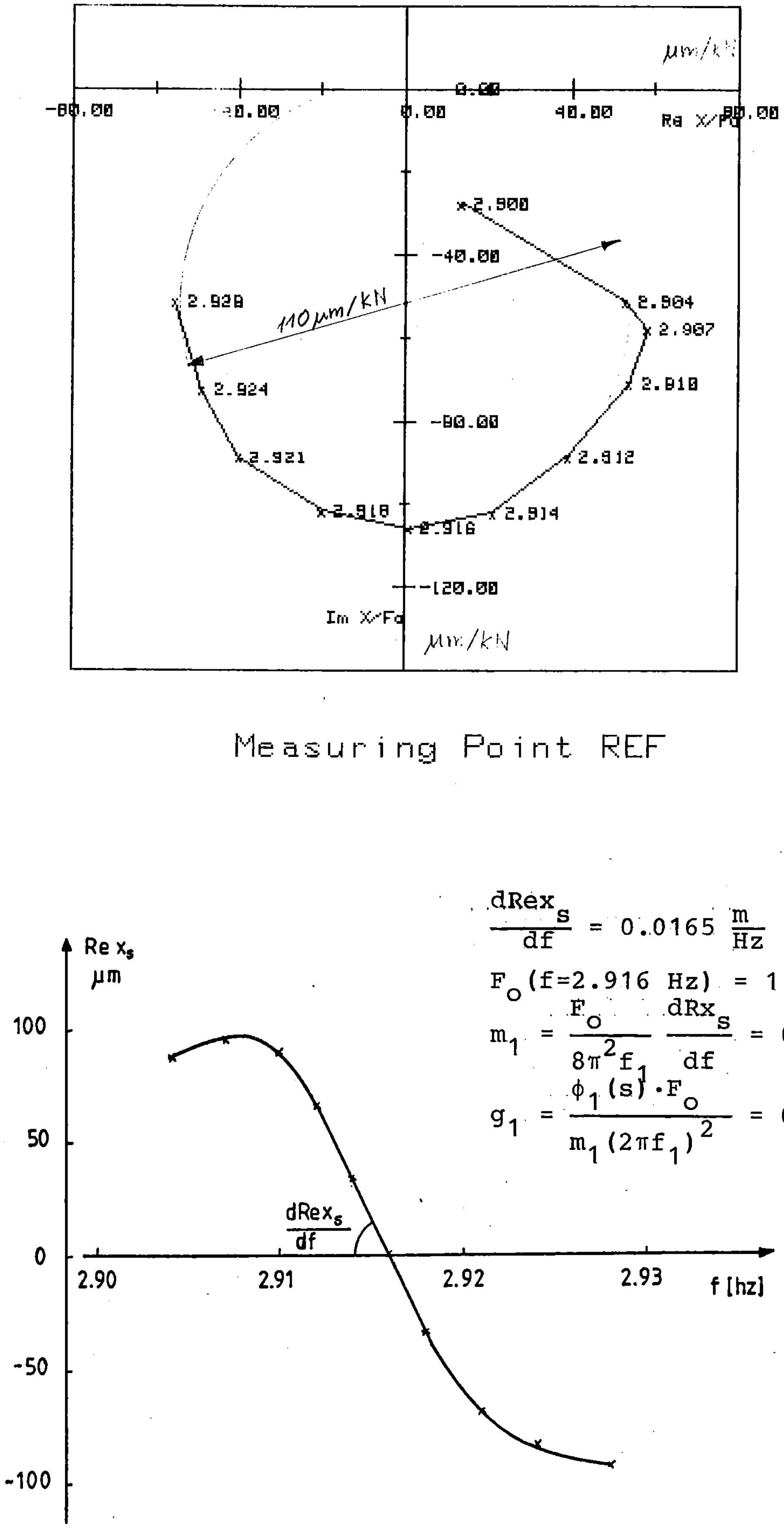


Fig. 17. Fourth natural mode of icebreaker SISU.



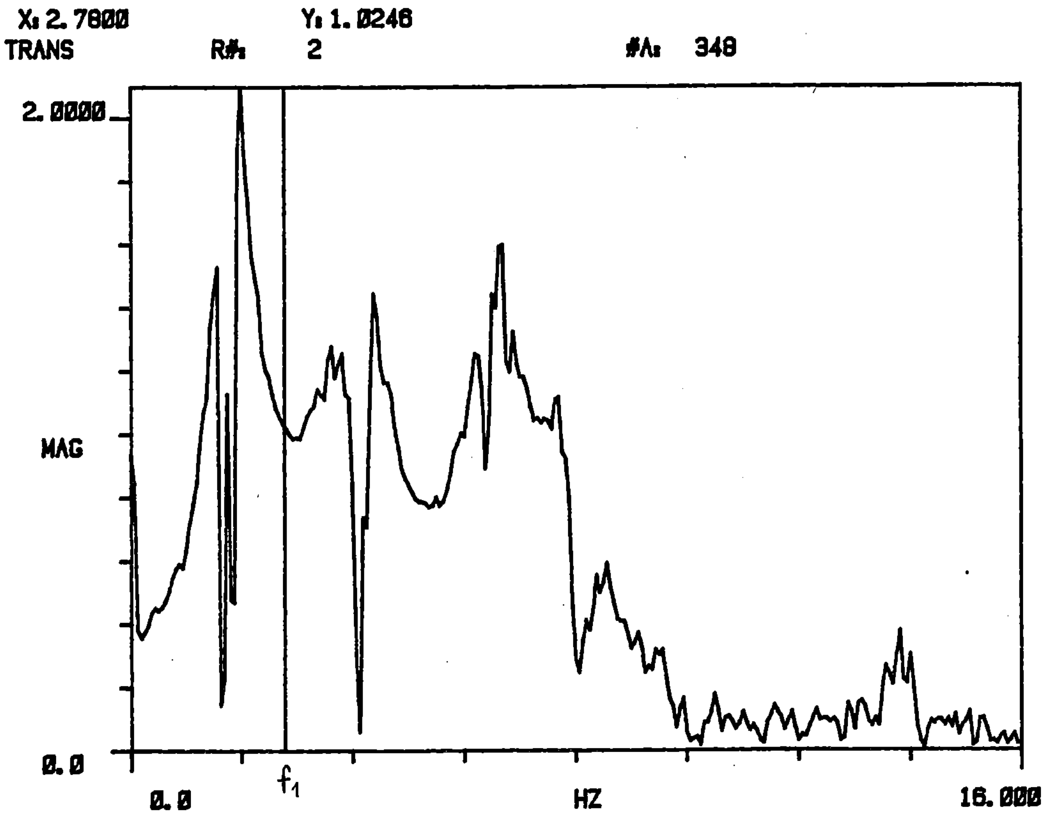
$$\frac{dRex_s}{df} = 0.0165\ \frac{m}{Hz}$$

$$F_o(f=2.916\ Hz) = 1.649\ kN$$

$$m_1 = \frac{F_o}{8\pi^2 f_1} \frac{dRx_s}{df} = 0.118\ kg\ m^2$$

$$g_1 = \frac{\phi_1(s) \cdot F_o}{m_1 (2\pi f_1)^2} = 0.0075$$

Fig. 18. An example of evaluating the generalized parameters (first natural mode).



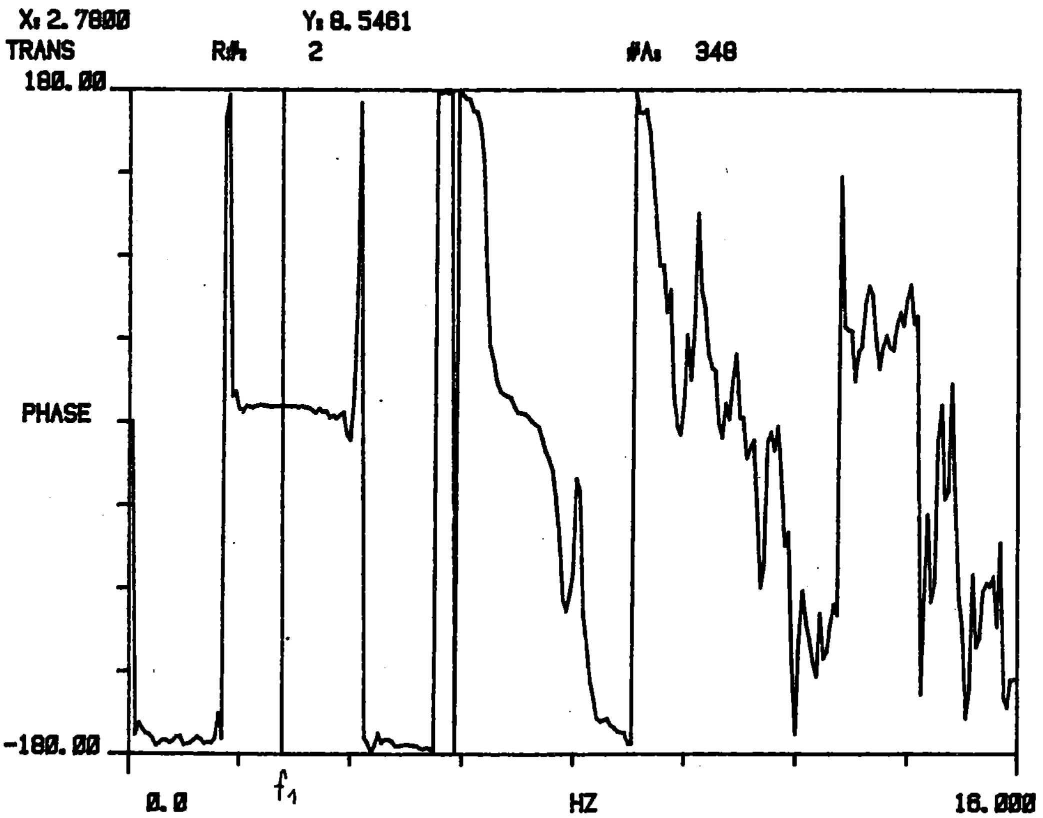


Fig. 19. Transfer function of the ice-induced vibratory response (point K7 vs. point REF).

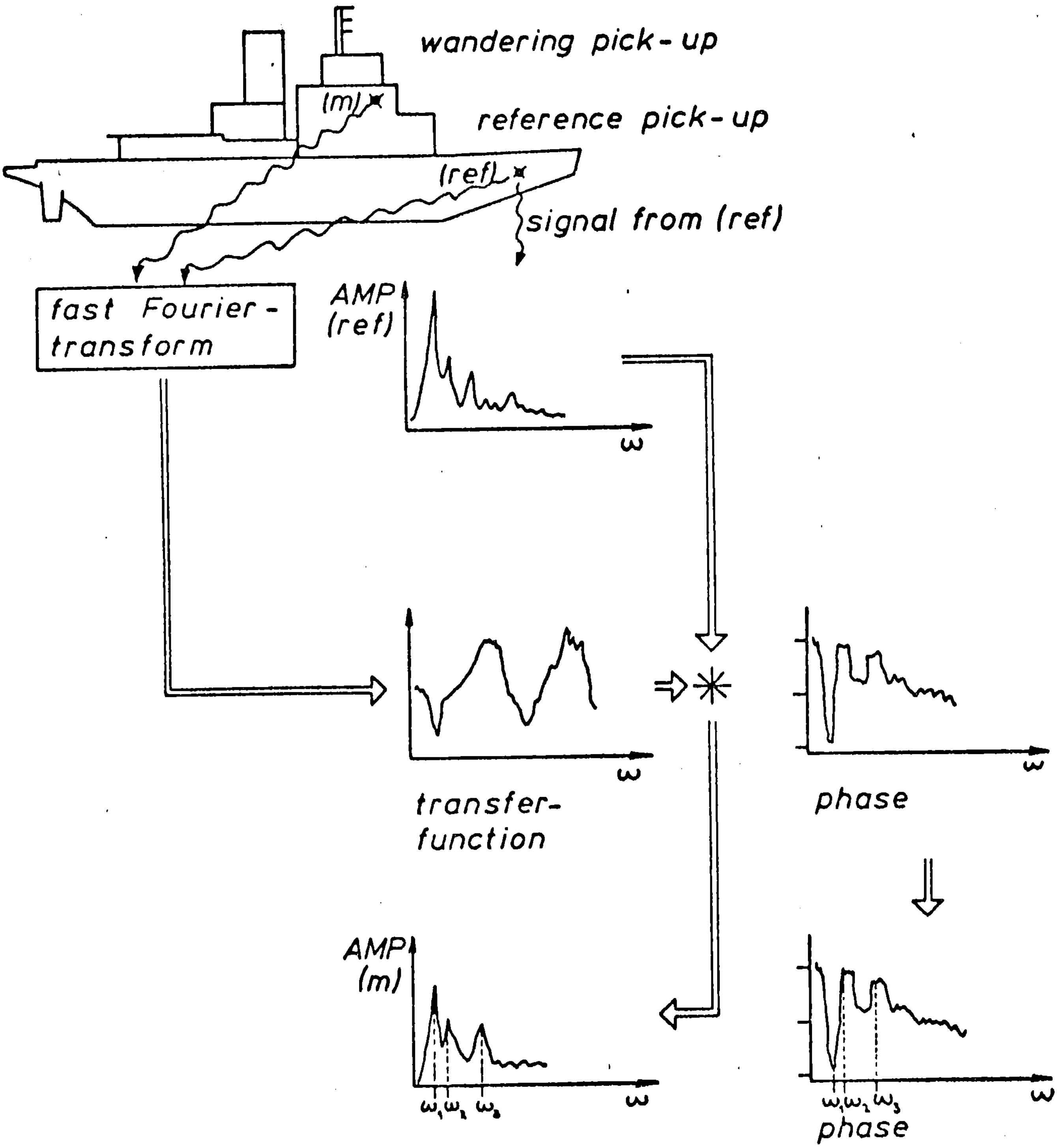


Fig. 2a A scheme of the analysis of ice-induced vibration.

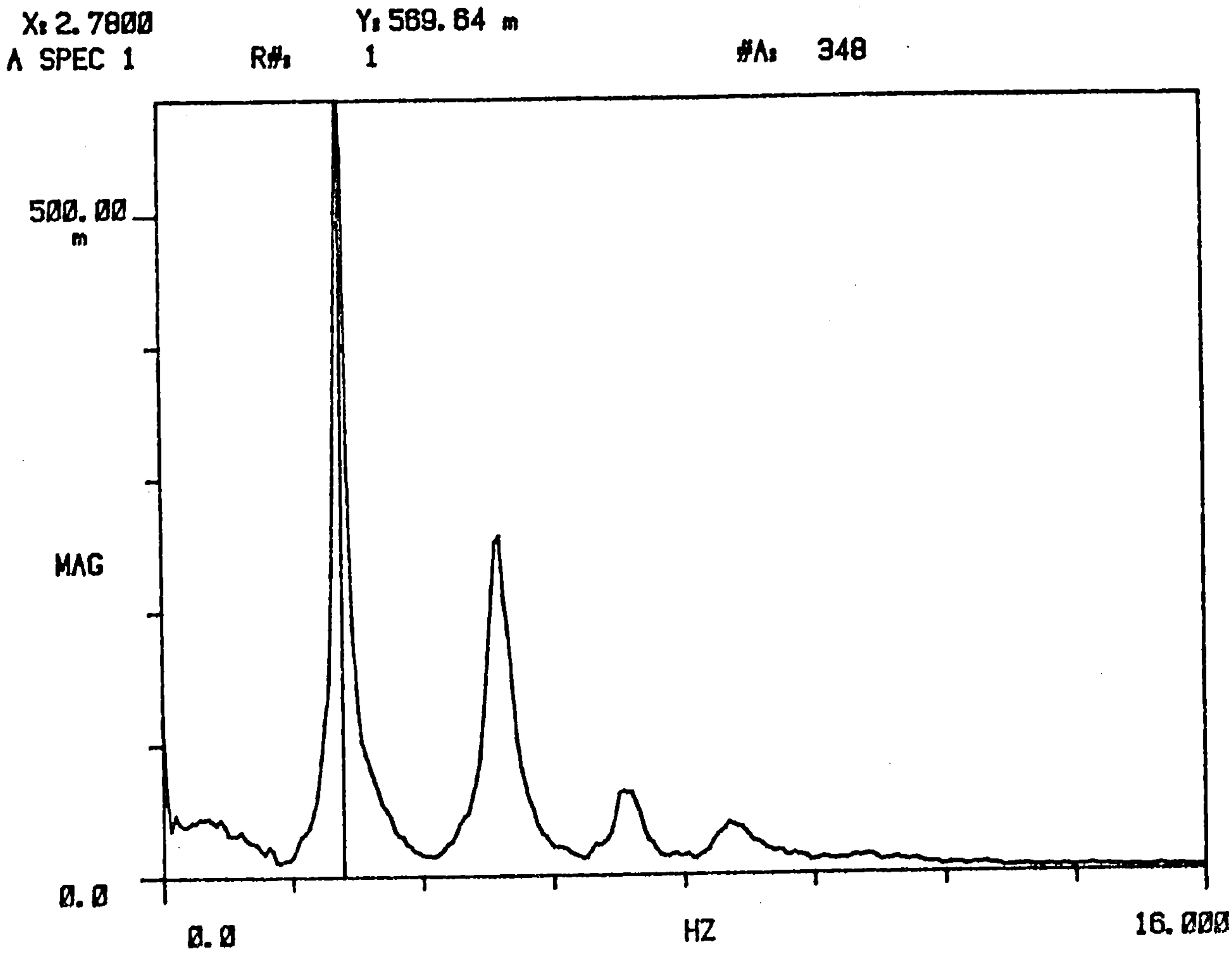


Fig. 21. Typical amplitude spectrum of the ice induced vibration.

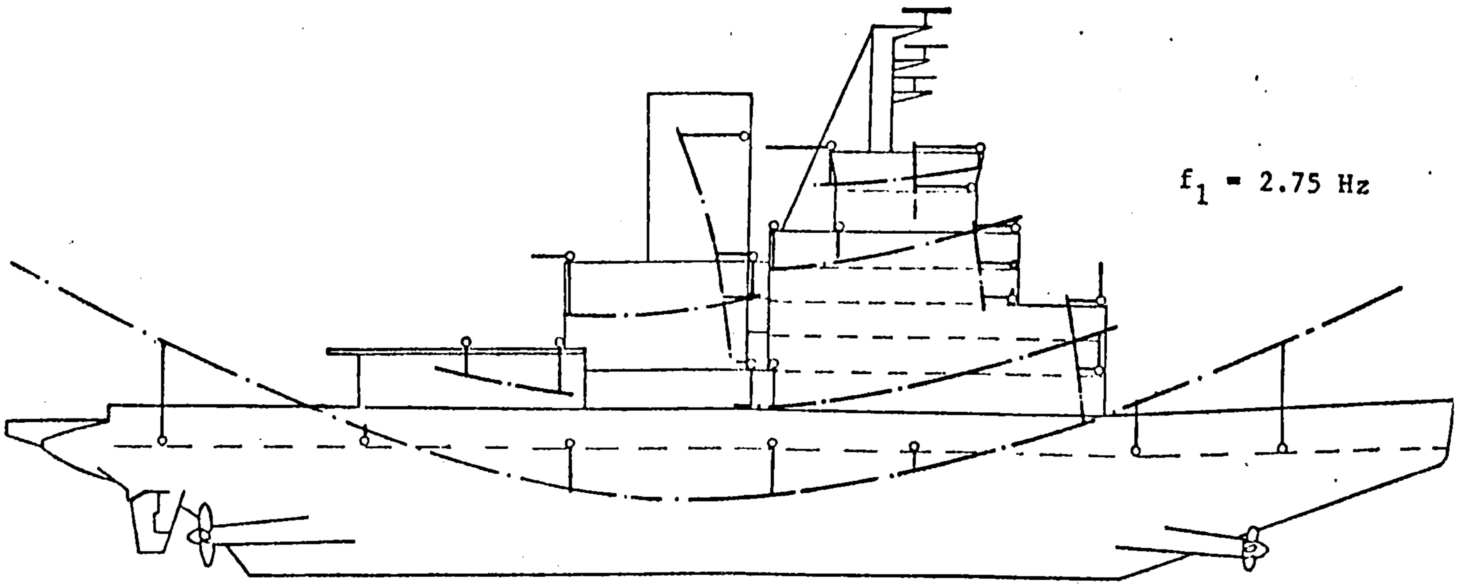


Fig. 22. First vertical natural bending mode of ship excited by ice-impacts.

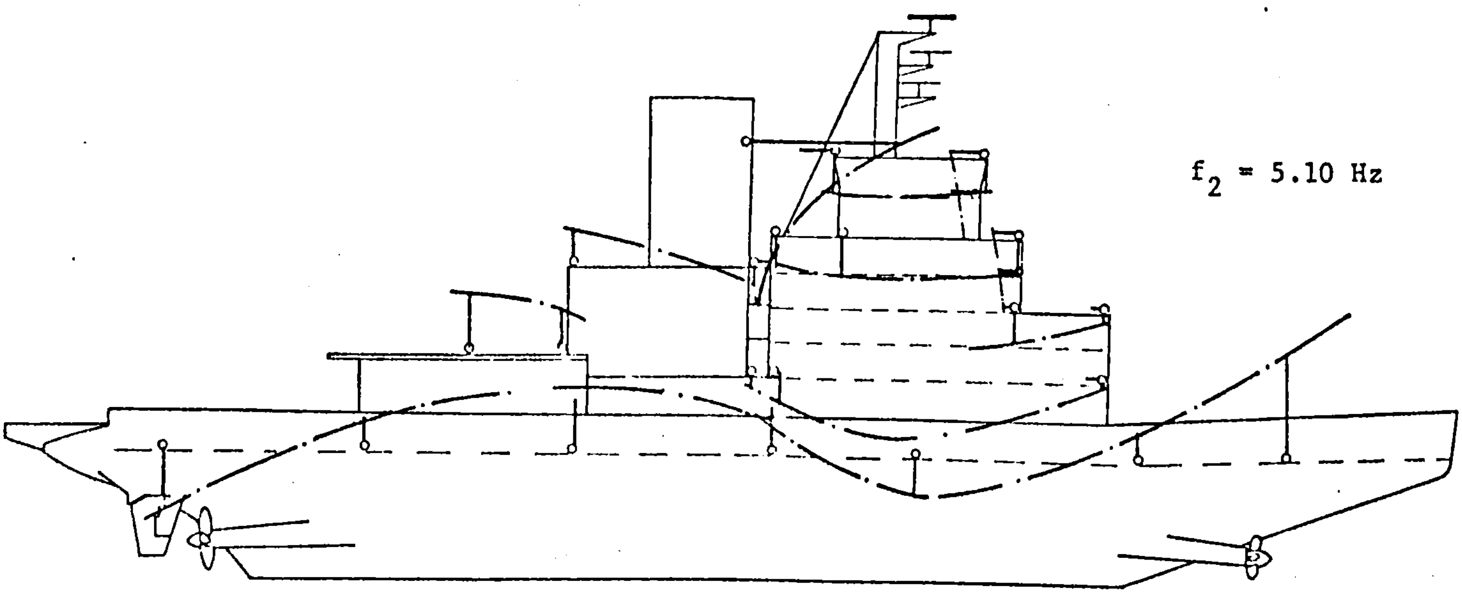


Fig. 23. Second vertical natural bending mode of ship excited by ice-impacts.

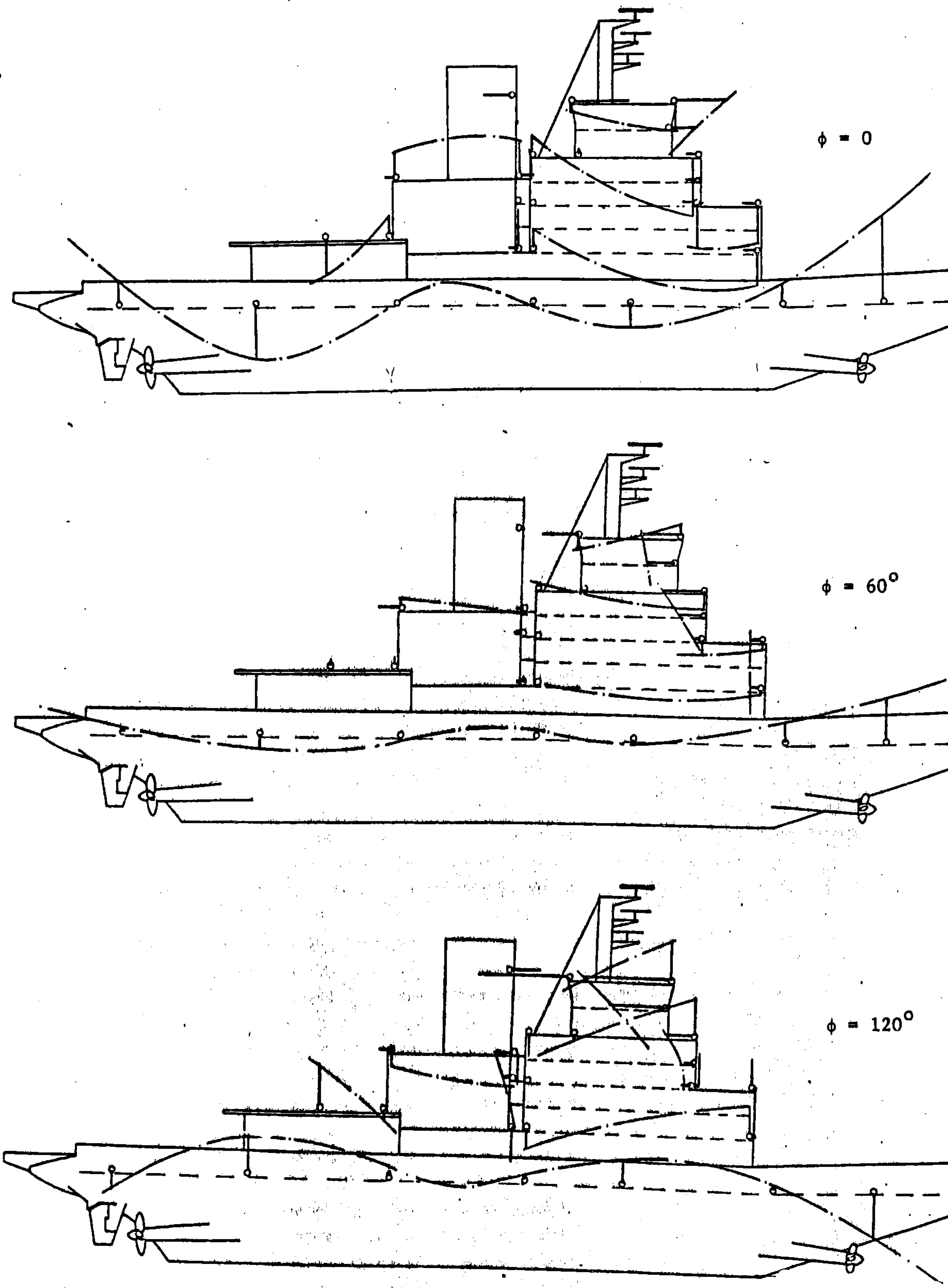


Fig. 24. Resonant vibration due to ice-impacts with frequency f_3 = 6.80 Hz.

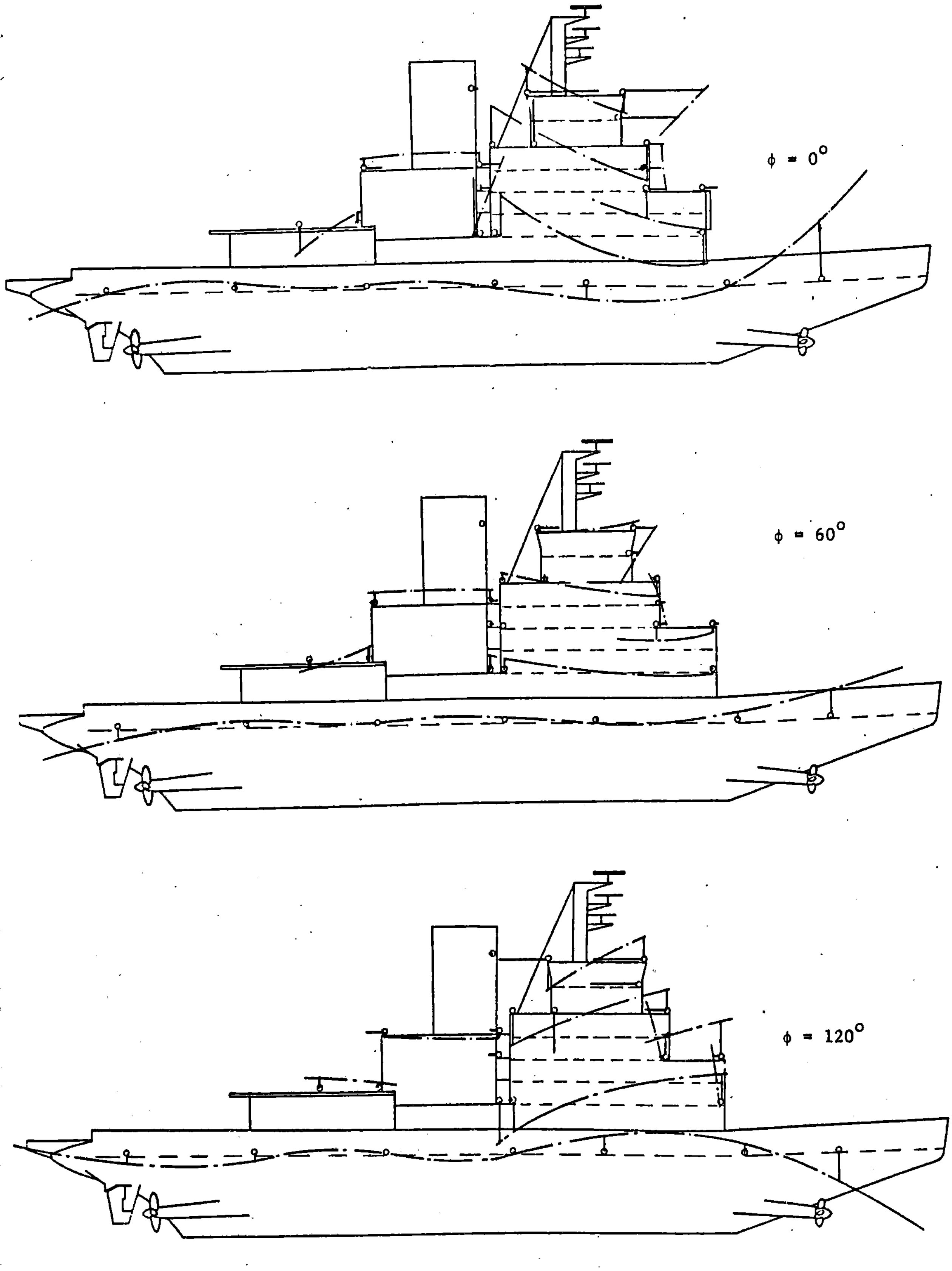


Fig. 25. Resonant vibration due to ice-impacts with frequency f_4 = 8.44 Hz.

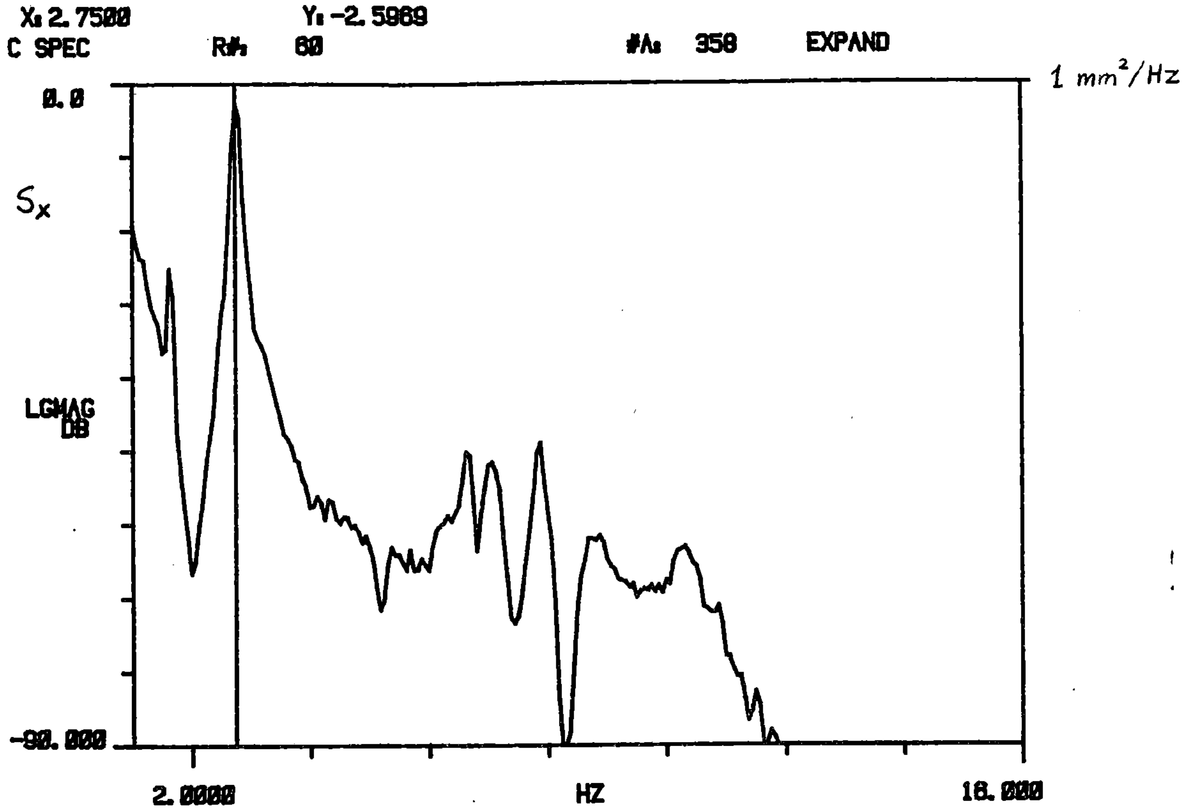


Fig. 26. Spectral density of the reponse measured at point KK1L.

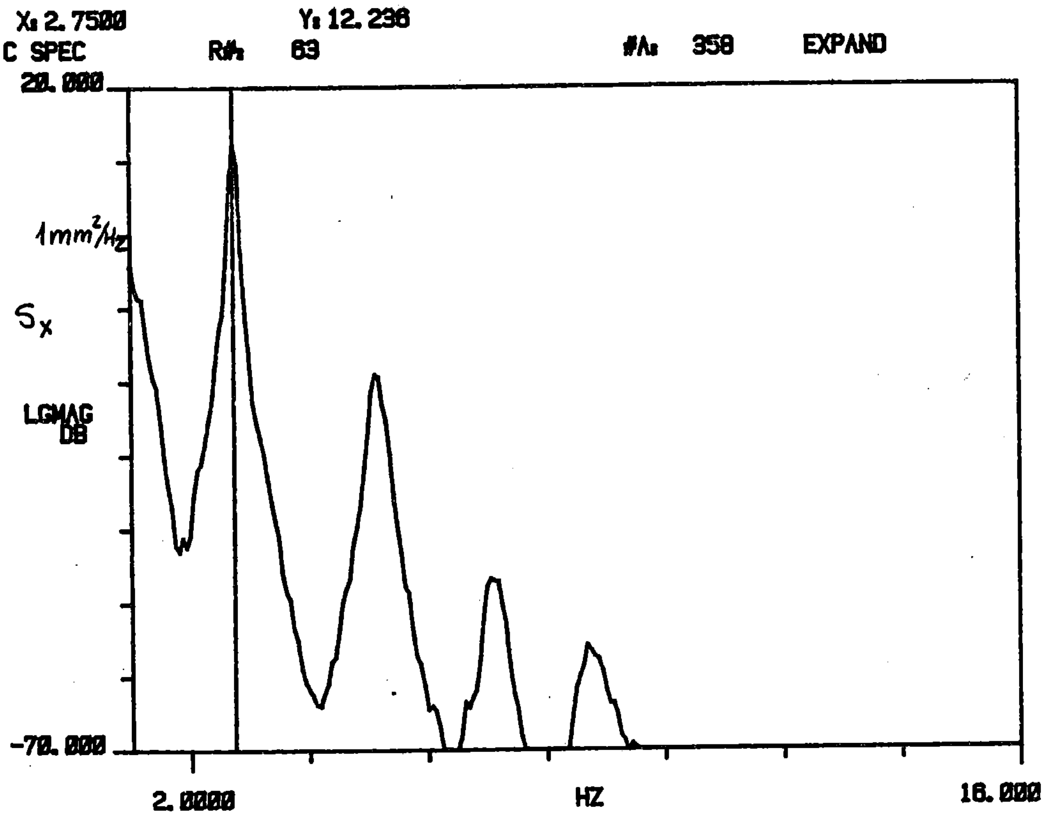


Fig. 27. Spectral density of the response measured at point R1.

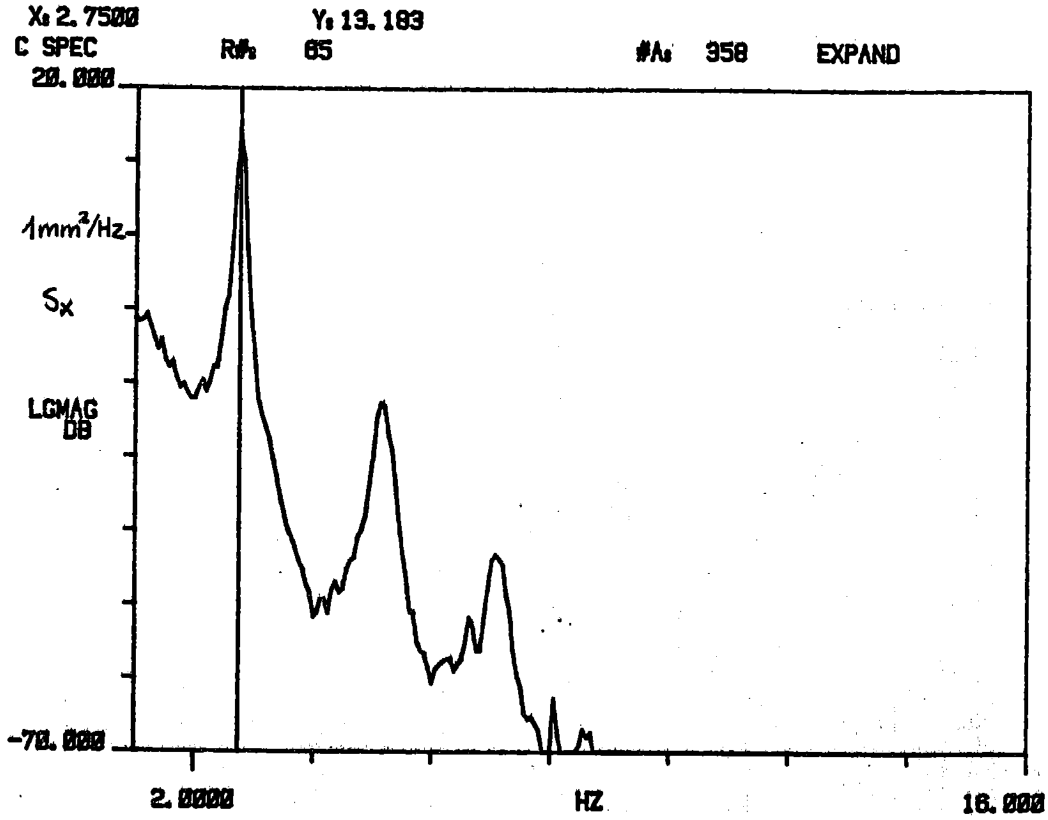


Fig. 28. Spectral density of the response measured at point K7.

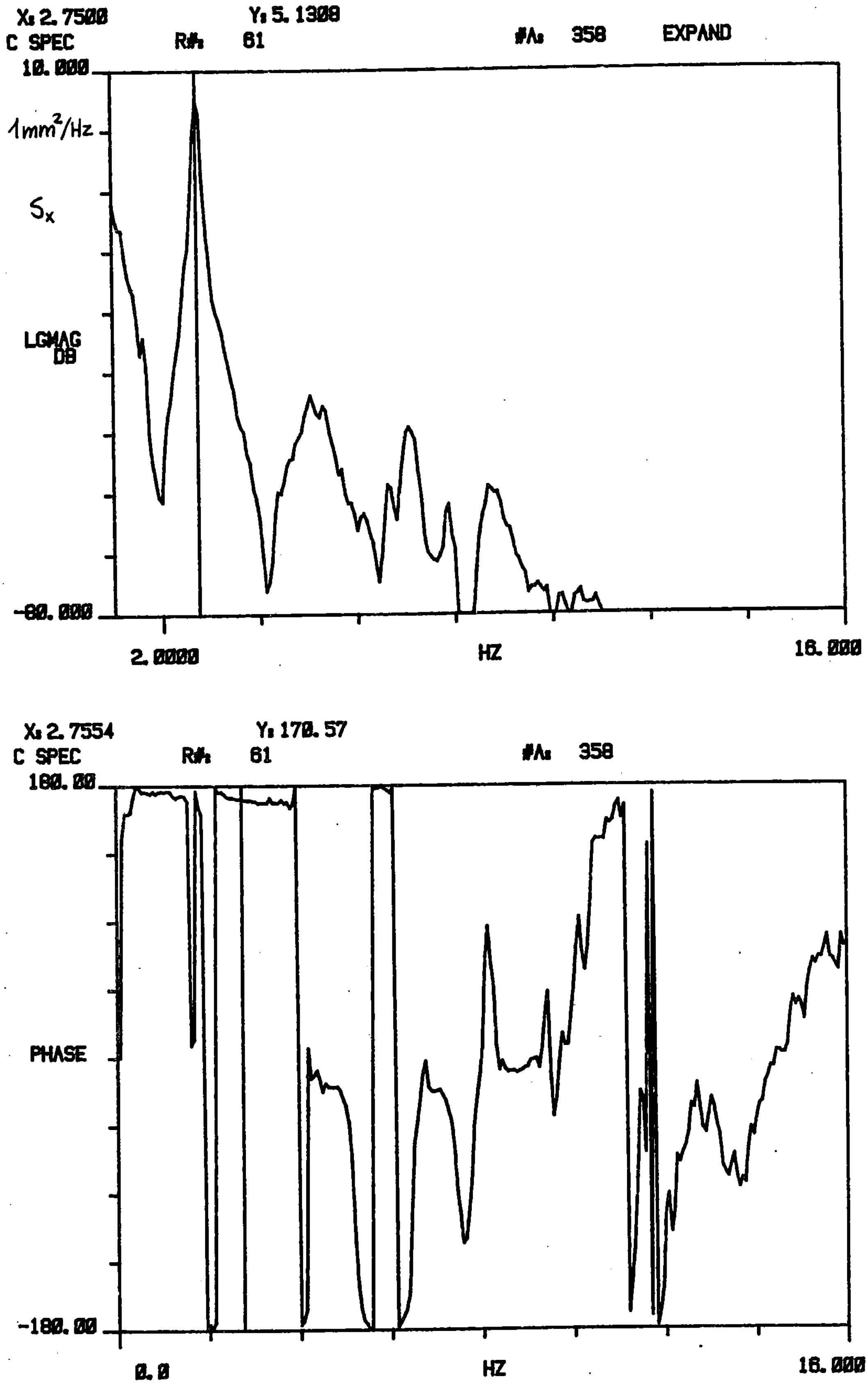


Fig. 29. Cross-spectral density of the response measured at points KK1L and R1.

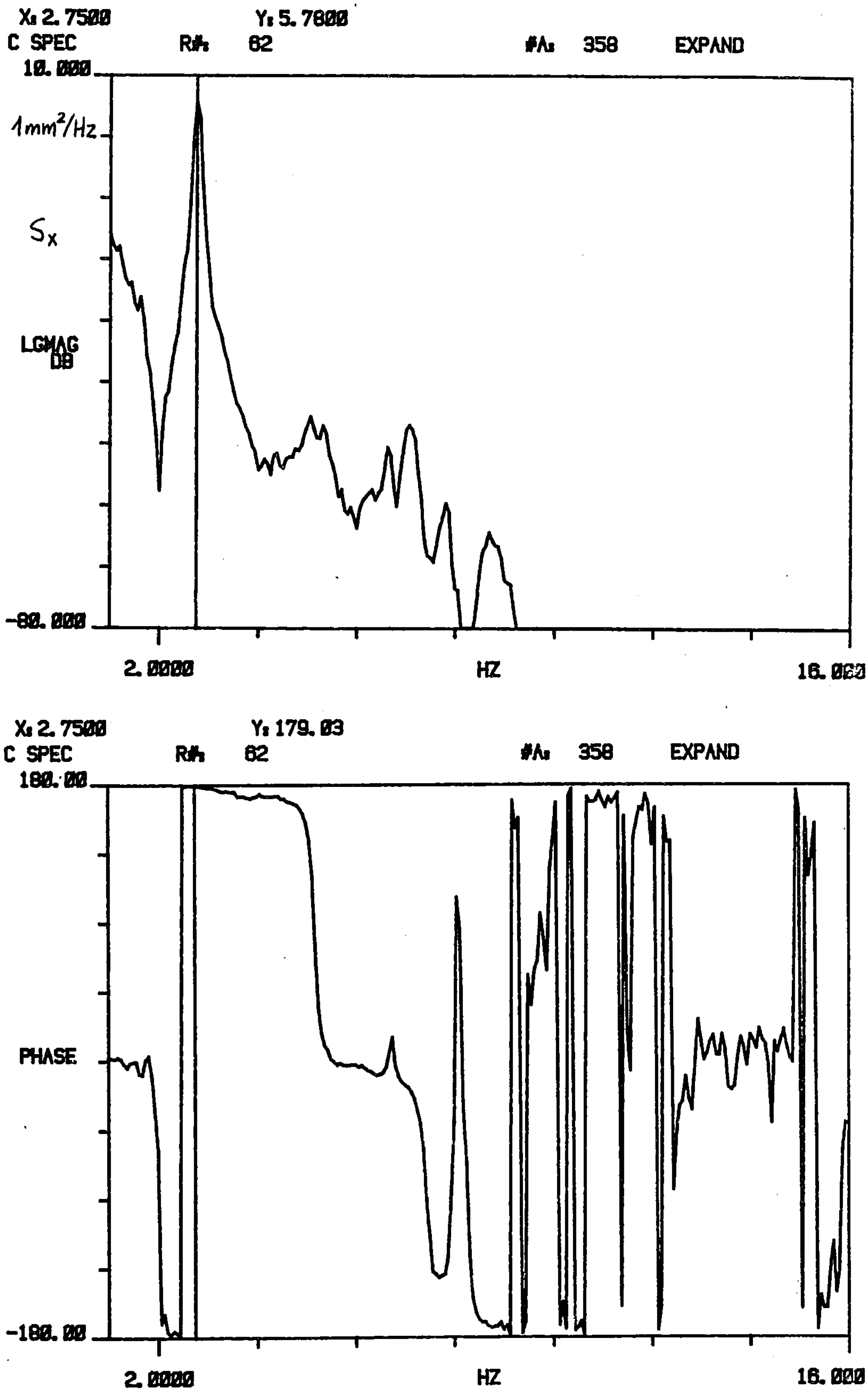
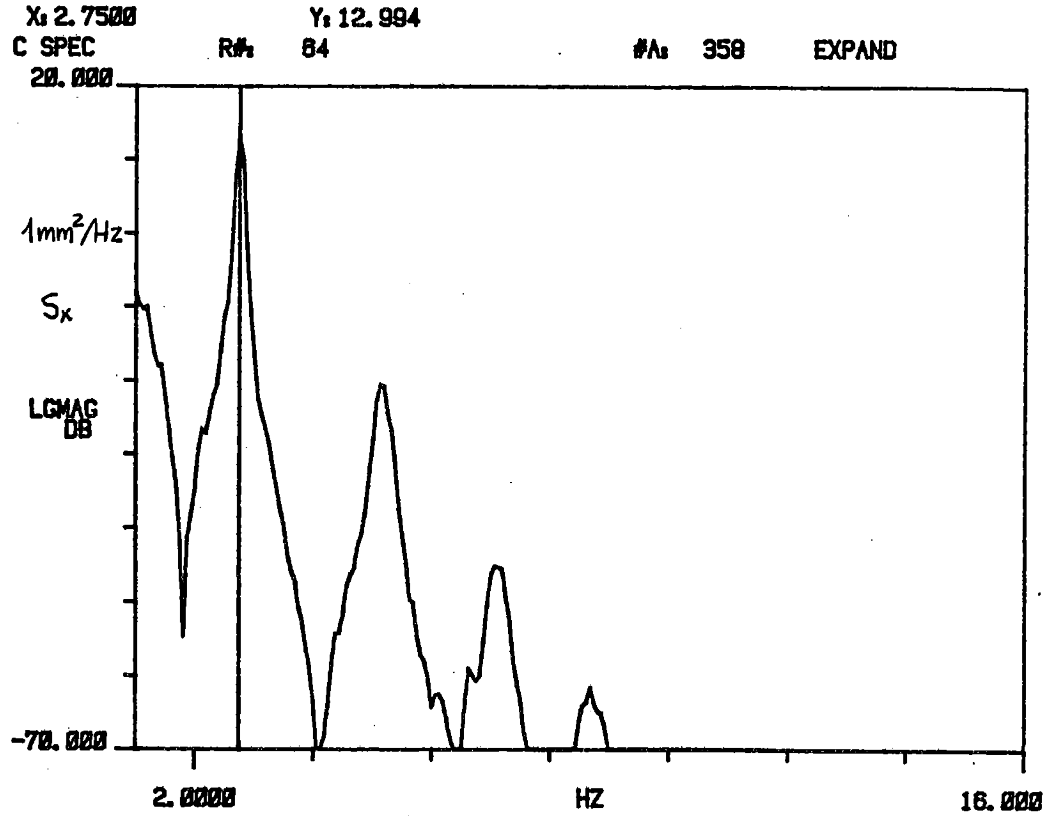


Fig. 30. Cross-spectral density of the response measured at points KK1L and K7.



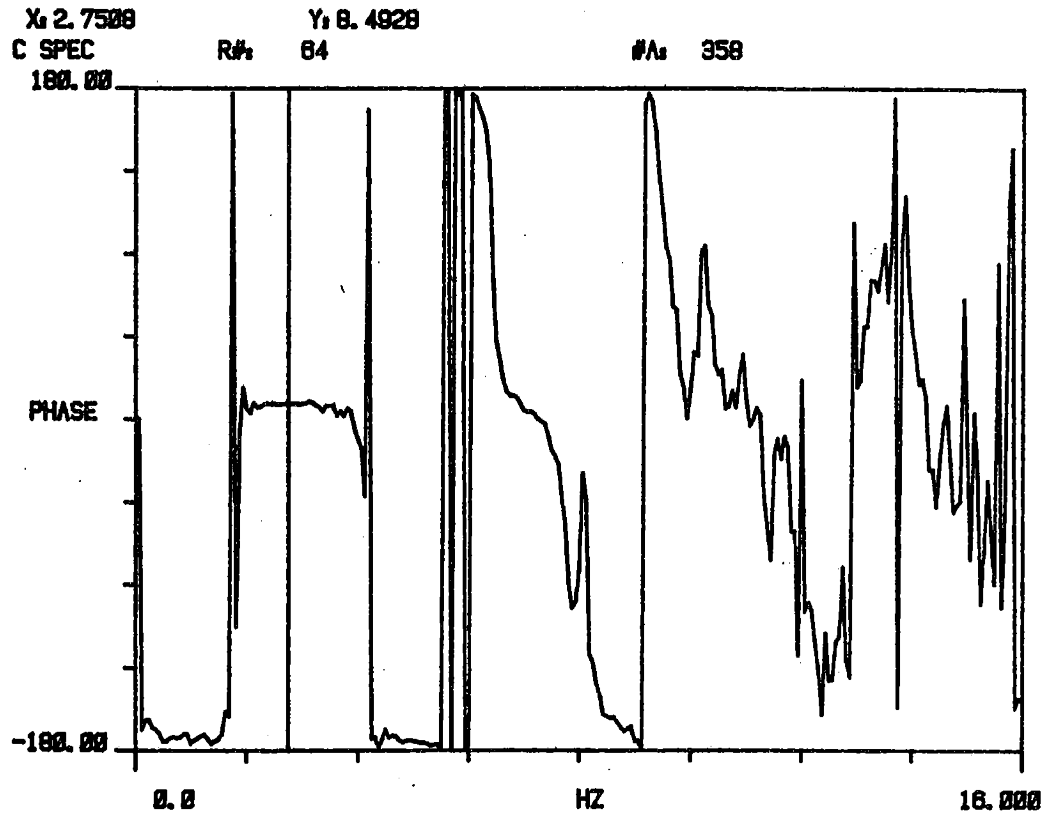


Fig. 31. Cross-spectral density of the response measured at points R1 and K7.

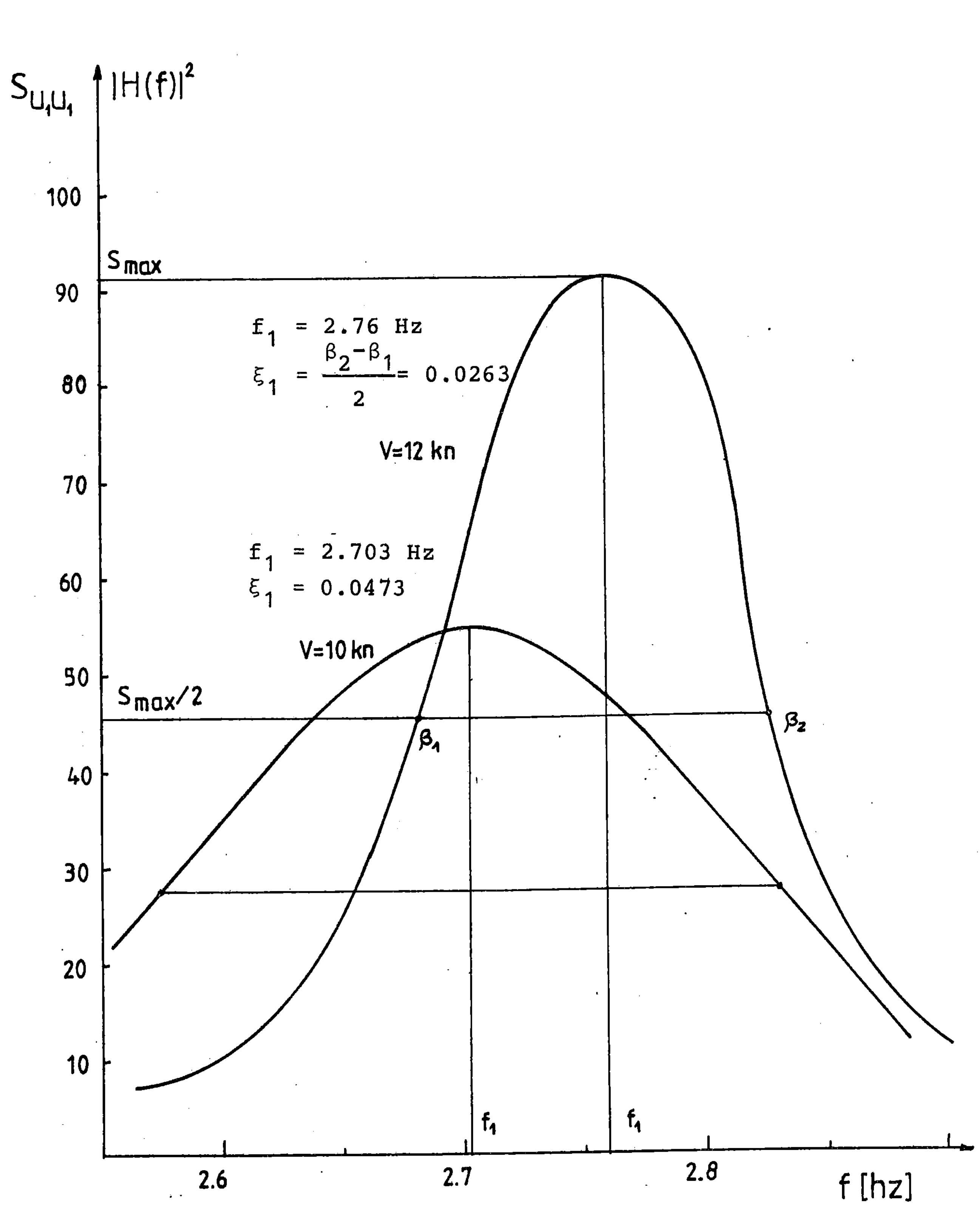


Fig. 32. Ice-induced vibratory response of SISU in terms of the spectral density of first generalized coordinate.

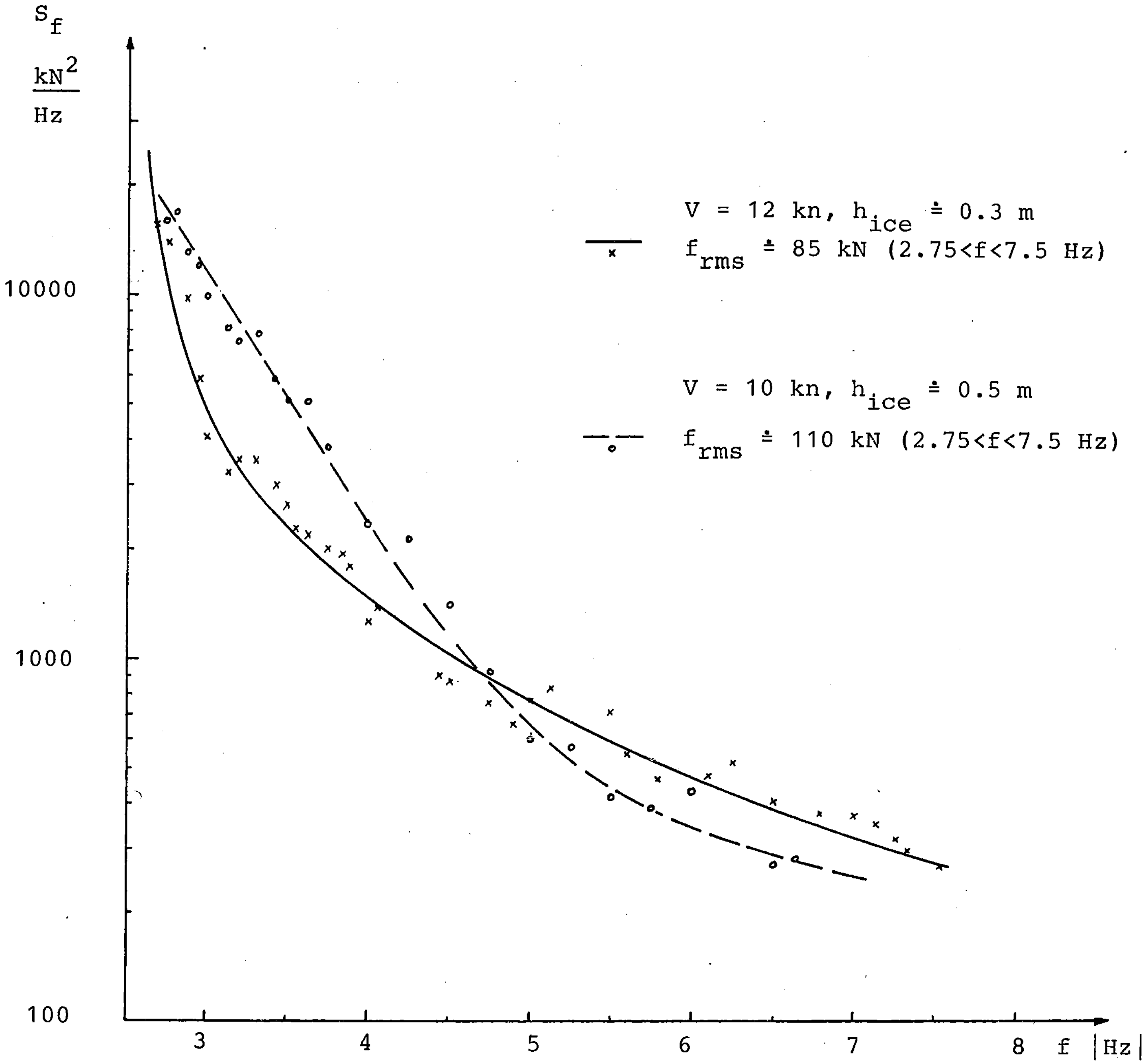


Fig. 33. Spectral density of the ice loads equivalent bow force for icebreaker SISU.