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DESIGN POINT IN ICE CLASS RULES

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FOREWORD

In its report no 71, the Winter Navigation Research Board presents the outcome of the project on design point in ice class rules. Ice class rules like the Finnish-Swedish Ice Class Rules, Russian Maritime Register of Shipping ice rules and International Association for Classification Societies harmonized ice class (PC-) rules each contain an explicitly given or implicitly used definition of the design point. The design point specifies what the allowable limit response of the structure is and how often this limit is reached. Various aspects of the design point are analysed and discussed in this report.

The Winter Navigation Research Board warmly thanks Professor Kaj Riska for this report.

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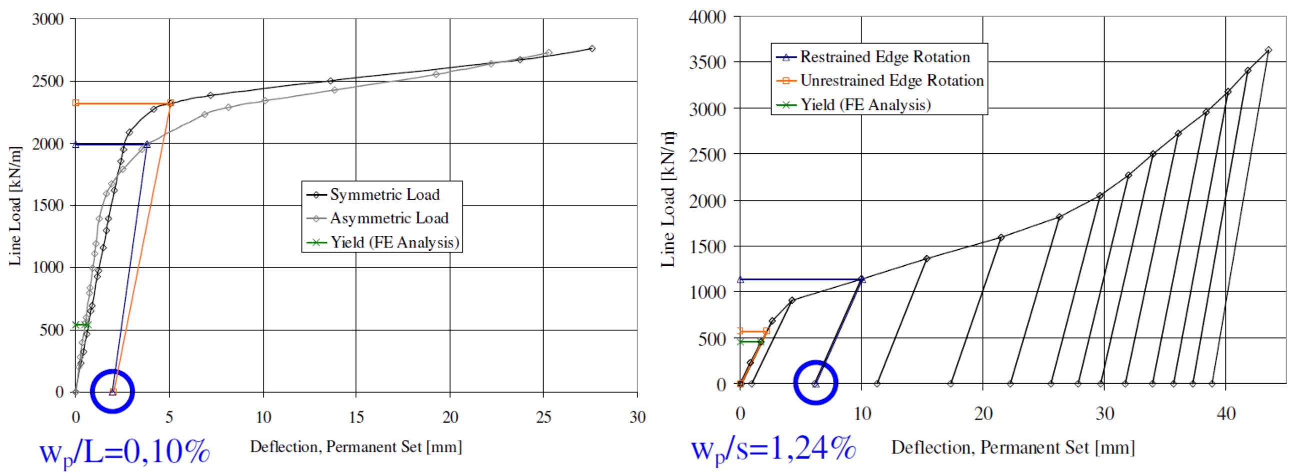

Hayward (2007)

14.3.2011 / Kaj Riska

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Consulting Naval Architects & Marine Engineers

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Cover figures: Deformation of a frame and plating under ice load taken from Hayward (2007).

1. INTRODUCTION

Ice class rules like the Finnish-Swedish Ice Class Rules, Russian Maritime Register of Shipping ice rules and International Association for Classification Societies harmonized ice class (PC-)rules each contain an explicitly given or implicitly used definition of the design point. The design point specifies what the allowable limit response of the structure is and how often this limit is reached. Allowable limit responses are often characterized by Ultimate Limit State (ULS), Serviceability Limit State (SLS), Accidental Limit State (ALS) or Fatigue Limit State (FLS), see e.g. NORSOK (2004). It is important to note that in the design point only the frequency and the limit state are free parametres; loading for example follows in principle from the intended operational spectrum and the frequency.

When knowledge about loading and description of the limiting structural response are available, the required (minimum) values of scantlings can be determined. A balance between the frequency of reaching the design response and the magnitude of this response is important – this balance is usually determined by the risk level to be achieved. If the allowable response is given as some amount of permanent deformation, the frequency of loading causing this should be low – similarly if the limiting response is stress up to yield limit, this can be allowed to occur more often. The balance of the frequency of reaching the allowable response and the definition of the allowable response is the topic of this investigation.

Knowledge about the structural response under the expected loading – local ice loads from first year ice in this case – is required for the evaluation of the design point. The knowledge of the loading includes quantities describing the load (pressure and the load patch dimensions) as well as the statistics of the loading. The ice load quantities are shortly described in this report and the assumptions to determine the ice load quantities made in the analysis are justified. The load statistics should ideally be derived from the encountered ice conditions in similarity of the wave bending loading derived from wave statistics but as this link between the load statistics and ice conditions is rather sketchy at the moment, measured ice loads from the chemical tanker Kemira from the Baltic are used.

The structural response investigated here includes only the response of the local structures i.e. the structural members of the shell of the ship hull. Thus the investigation is restricted to plating and frames. A further restriction is to investigate only transversely framed structures as knowledge about ice loading statistics for longitudinally framed structures does not exist. The structural response formulations used in the analysis are obtained from literature for both the elastic response and the plastic response.

The investigation reported here is to be seen as a synthesis between the ice loading and the structural response formulations, using the allowable response (limit state) as a parameter. Thus the research topic is neither the statistics (or other characteristics) of the ice loading in itself nor the response formulations. These methods are used as a means to an end i.e. to evaluate the balance between the frequency and severity of the response. Thus the ice load and response formulations leave much to be desired. Especially the ice load parameters, the influence of the ice conditions encountered on ice load statistics and plastic response formulations for frames could be addressed more in depth.

2. METHODOLOGY

If the loading and the strength of the structure are of stochastic nature, these must be described by statistical distributions. When this is the case, the design can be carried out using (either the approximate or exact) distributions for loading and structural strength and then calculating the probability of failure (a good synopsis of these methods in given e.g. in Jensen 2001). In order to define the approach in this investigation, a short description of the formulations relating to the probability of failure (denoted as P_f) is given following Jensen (2001).

Let the load and structural strength be described by a variables denoted as q and Q, respectively, and let further the distributions (probability density functions, pdf) of these be $f(q)$ and $s(Q)$. Then the limit state function is $L(q,Q) = Q - q$ and the allowable (safe) area is $L(Q,q) \geq 0$. It will be assumed that the load may be described by a single load parameter q when in fact there might be several load parameters like the contact ice pressure and the load patch dimensions. This is discussed in Chapter 2.1 where it is explained that the load length and height are defined as equivalent quantities calibrated by the measurements.

2.1 Description of Ice Load

The ice load for the structural design purposes is commonly assumed to be described by uniform ice pressure p on a rectangular load patch of height h and length l. Thus the total force is $F = p \cdot h \cdot l$. The ice load measurements that are described in more depth in Chapter 3 give the ice load acting on one frame which is $F = p \cdot h \cdot s$ (assuming naturally that the load length is more than one frame spacing s). This load description suggests a useful load quantity; that of the line load $q = p \cdot h$. This line load can be measured and for this reason is used in the subsequent analysis. The other quantities from the ice load measurements to be used in this analysis are the frame stress on top of the frame flange σ_{FR} and the plate stress in horizontal direction σ_{PL}. The stresses will be used to calibrate the formulation of the structural response.

The load length and load height influence the response of plating and frames. The frame bending response is not sensitive to load height if the load height is much smaller than the frame span (which is the case mostly). For plating the load height is much more important parameter. Plate response is also sensitive to the shape of the pressure distribution. It has been suggested that the flexibility of the plate in comparison with frames induces a pressure drop in the middle of the frame spacing, see e.g. Uto (2000). This pressure shape effect and the load height are described when deriving the response formulation in Chapter 2.3.

The load length has been observed to influence the average line load; the longer the load length the lower the average line load on the whole length, see the discussion by Riska in Kujala, ed. (2007). At smaller load lengths there is a cut-off as the line load is assumed not to increase without limit at smaller length; this cut-off is about two frame spacings (600 mm). If the load length is smaller than about 2s, the frame response is influenced by the support of the adjacent frames; some load is shed to the adjacent frames or, in other words, the adjacent frames support the frame in question. If the load length is small, this effect must be taken into account in formulating the frame response, this is described in chapter 2.3. The ice load and structural response are to be calibrated by measured ice load and the measured stresses in the frames and plating. Only the bow frame is investigated and the midbody and stern strength formulation is assumed to be similar.

2.2 Formulation of Ice Load Statistics

The local ice loading process on the ship shell consists of separate peaks; see Fig. 1. Each of the peaks represents an impact with an ice edge that consists of separate cusps. This ice edge shape is formed by the breaking pattern. When the ship hits an ice cusp, the ice edge is first crushed and the contact force increases with increasing contact area. The maximum force is determined by the bending strength of the ice cover. At any fixed location at the hull, the loading process consisting of separate peaks seems quite stochastic. The stochastic ice loading process is to be described by statistical distributions - and it is natural to assume the ice load distribution to be the distribution of the load peaks. Often, however, the maximum load values are described by the maximum value observed within some selected time period. This period is for most of the Baltic ship ice load measurements that have been carried out either 12 h or 24 h.

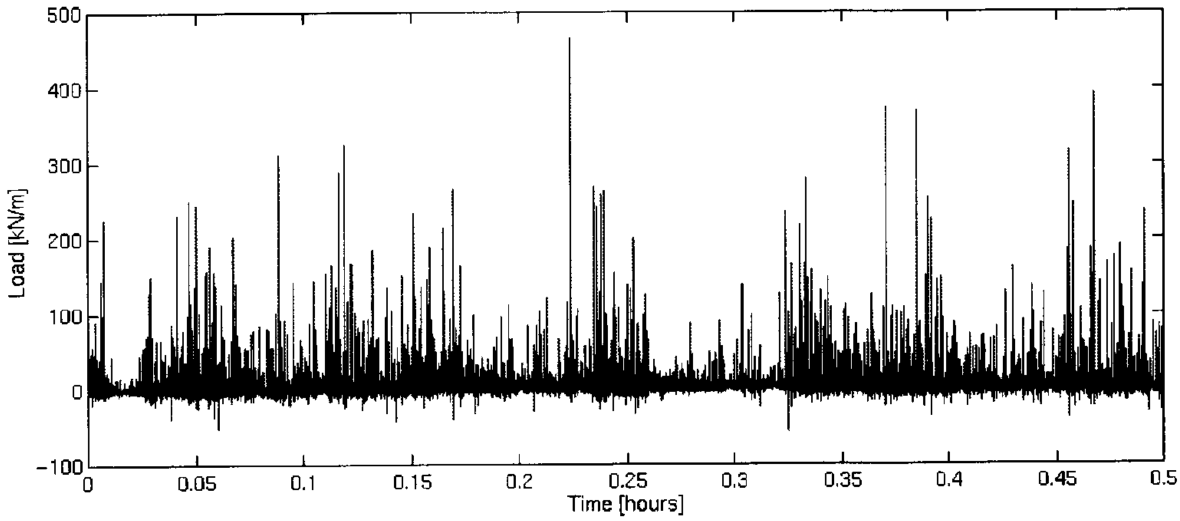


Fig. 1. A measured time series of the ice load on one single frame of the MT Uikku (Lensu 2002).

Before describing the probabilistic design approach, the ice load distributions to be used in this study are described briefly. If the probability distribution function (pdf) of the ice load peaks is denoted as $f_P(q)$, the corresponding cumulative distribution as $F_P(q)$ and the observed frequency of the peaks as ν_P, then the cumulative distribution of the maximum peak in the time period T is

$$F_T(q) = [F_P(q)]^{\nu_P T}, \tag{1}$$

assuming the peaks to be statistically independent. This expression can in principle be extrapolated to any time period T, including the ship’s life time. This kind of extrapolation is, however, somewhat extreme and at least the basic data in determining the $F_P(q)$ and ν_P should reflect the different ice conditions encountered during the ship’s life time.

As was mentioned above, often only the maxima from time period T_0 are available. These maxima can be used to determine the cumulative distribution $F_{T_0}(q)$ which can then be extrapolated to ship’s life time T_L as

$$F_{T_L}(q) = \left[F_{T_0}(q)\right]^{T_L/T_0} = \left[F_P(q)\right]^{\nu_P T_L} .$$

The extrapolated distributions (from $F_P(q)$ to $F_T(q)$ or from $F_{T_0}(q)$ to $F_{T_L}(q)$) can be estimated using so called asymptotic Gumbel distributions. These distributions capture the tails of distributions better than the original 'parent' distribution. The tails of probability distributions are important for design. Depending on the nature of the 'parent' distribution $f_P(q)$, three different Gumbel distributions, type I, II and III, have been derived. Here only the simplest, Gumbel I distribution is used. The cumulative function of this distribution is

$$G(q) = e^{-e^{-c(q-u)}} , \tag{2}$$

where c and u are the parameters of this distribution. If the mean (q_{av}) and the standard deviation (σ_q) of the measured set $\{q_i\}$, i=1,...,N are known, then the Gumbel I distribution parameters can be estimated using the moment method as

$$c = \frac{\pi}{\sqrt{6} \cdot \sigma_q}$$

$$u = q_{av} - \frac{\gamma}{c}$$

where γ is the Euler constant (value 0.5772...).

The asymptotic distribution can be used to derive a useful quantity that will be used subsequently i.e. the return period T. This is the time period in which the maximum value among peaks will exceed the most probable value of the distribution g(q) , $g(q) = \frac{dG(q)}{dq}$. The return period is defined as

$$T = \frac{1}{\nu_P} \frac{1}{1 - G_P(q_T)} \tag{3}$$

where the subscript T has been used to note that this is the most probable load value corresponding to the return period T. This expression will be used in the present investigation; when the $G(q)$ is Gumbel I distribution of maxima each from a time period of length T_0, the most probable load for the time period T is

$$q_T = u - \frac{1}{c} ln(-ln(1 - \frac{1}{\nu_P \cdot T})) . \tag{4}$$

Instead of extrapolating the peak distribution the basic i.e. parent distribution could be the measured maxima from some time period (subscript $P \rightarrow T_0$).

If the measured sample contains measured 12 h maximum loads from a whole winter, and preferably several winters, then the data base is quite robust and can be considered to represent the ice loading of this kind of ship in the general ice conditions in the area. One such data base exists, from chemical carrier Kemira operating in the Baltic, see Muhonen (1992). These data cover years 1985 – 1991, during this time several different (mild as well as severe) winters were experienced. That ship specific data represents just one hull form, one ship size and one operational profile is the major lack of this kind of data base. The Kemira data consists of maxima recorded for each 12 h period. The data used in this analysis is described in chapter 3.

2.3 Structural Response Formulation

The aim of structural response formulation is to derive a relationship between the limit response, scantlings and the load. Here only transversely framed hull structure is investigated. The limit response that is investigated includes both elastic and plastic cases. More exactly the limit states that are investigated include the cases shown in Table 1.

Table 1. Definition of the limits states for plating and frames.

Limit state (label)	Plating	Frames
Elastic (Y)	Stress reaching the yield stress σ_y somewhere in the plate.	Stress reaching the yield stress σ_y somewhere in the frame.
Plastic (P)	Stress distribution reaching full plasticity somewhere in the plate. Permanent deformation still zero.	2-hinge formation at the frame supports.
Ultimate (U)	Permanent deformation (w_P) reaching a specified value.	3-hinge formation at the frame supports and middle.

The reason why the permanent deformation is specified for plating but not for frames is that no easy formulations are available for frame deformation. Some formulations exist like the one given in NORSOK N-004 (Appendix A, section 3, eq. A.3.28) but it was decided to use for this scoping study simple 2- and 3-hinge formulations. These also ignore shear deformations. The plastic reserve due to membrane stresses is large for plating, and simple formulations for the permanent deformation of patch loaded plates exist (see for example the deformation curves of a frame and plating on the cover).

Several studies have been carried out investigating the plastic and ultimate response of ship hull structures under ice loads – one of the first ones is Johansson (1967). The other studies include studies in the project SAFEICE (Kujala, ed. 2007), and damage studies Varsta et al. (1978), Ranki (1986), Hayward (2001), Daley (2002a,b), Valkonen (2006) and Kaldasaun (2010). Development of formulations for elastic and plastic response of plating and frames would be a major topic in itself; here only some formulations are selected to be used in investigating the design point.

To calculate the elastic response of plating the formula developed for the Finnish-Swedish ice class rules (Trafi 2010) is used. The relationship between loading q and load height h, plate response i.e. stress σ and scantlings is:

$$q_{PL} = p_{PL} \cdot h = C_f \cdot p \cdot h = C_f \cdot q = \frac{9}{4} \cdot \left(\frac{t}{s} \right)^2 \frac{\sigma \cdot h}{1.3 - \dfrac{4.2}{(h/s + 1.8)^2}}, \quad (5)$$

where s is frame spacing and t plate thickness. The equivalent pressure on plating is denoted as p_{PL} which is the uniform pressure at load height h that gives the maximum stress σ. The reduction factor C_f is attributed in the rules to the pressure drop at the mid frame spacing. Its value in the rules is $C_f = 0.75$. This formulation is used as it has been calibrated extensively when developing the rules in early 1980's. When the stress reaches yield, the corresponding pressure is denoted as p_Y. The question of what load height and reduction coefficient to use is tackled in chapter 4.1.

The plastic response formulation of plating is taken from Hayward (2001). Hayward (2001) conducted a large set of FE-calculations of plastic response of plating and the results are deemed most reliable and robust to be used here. The line load to cause a fully plastic response with no permanent deflection ($w_P = 0$) is thus described by

$$C_f \cdot q_P = C_f \cdot p_P \cdot h = \frac{12 \cdot t^2 \cdot h \cdot \sigma_y}{s^2 \left(\sqrt{3 + \left(\frac{s}{L} \right)^2} - \frac{s}{L} \right)^2} \cdot \frac{1}{0.6701 \cdot \left(\frac{h}{s} \cdot \left(\frac{s}{t} \right)^{0.2} \right) - 0.1330 \cdot \left(\frac{h}{s} \cdot \left(\frac{s}{t} \right)^{0.2} \right)^2}, \tag{6}$$

where σ_y is the yield stress. The line load to cause the ultimate response is given as

$$q_U = q_P \cdot \left[1 + \frac{w_P^2}{3 \cdot t^2} \left(\frac{\zeta_0 + (3 - 2 \cdot \zeta_0)^2}{3 - \zeta_0} \right) \right] \text{ when } w_P/t \leq 1 \tag{7}$$

$$q_U = 2 \cdot q_P \cdot \frac{w_P}{t} \left[1 + \frac{\zeta_0 \cdot (2 - \zeta_0)}{3 - \zeta_0} \cdot \left(\frac{t^2}{3 \cdot w_P^2} - 1 \right) \right] \text{ when } w_P/t > 1, \tag{8}$$

where the permanent deformation is w_p and

$$\zeta_0 = \frac{s}{L} \cdot \left(\sqrt{3 + \frac{s^2}{L^2}} - \frac{s}{L} \right).$$

The corresponding equations for especially the plastic frame response are, as mentioned earlier, more complicated. The different boundary conditions of continuous beams and end brackets cause variation. Also if only bending deformation is taken into account, the formulation is often too simple as the shear deformation should also be taken into account. There does not exist a clear formulation for inclusion of shear deformation, and as there is quite large uncertainty in describing the ice loads, the frame response is obtained in this study by describing only the simple bending response.

When a frame is considered as a clamped beam (span L) and loaded at the midspan by a patch load of height h with the load intensity $p \cdot s$, then the bending moment at the supports is

$$M = \frac{(3 \cdot L^2 - h^2) \cdot C_l \cdot p \cdot h \cdot s}{24 \cdot L}, \tag{9}$$

where the coefficient C_l takes into account the support of the adjacent frames. The value of this coefficient depends on the load length as well as the plate thickness and frame spacing. The value of the coefficient must be one if the load length is large i.e. covers more than, say, three frame spacings. The value for the coefficient C_l is derived in Chapter 4.3.

The displacement at the middle of the beam corresponding to (9) is

$$\delta = \left(2 \cdot L^3 - 2 \cdot L \cdot h^2 + h^3\right) \cdot \frac{C_l \cdot p \cdot h \cdot s}{384 \cdot E \cdot I}, \tag{10}$$

where E is the Young's modulus and I section moment of inertia.

When the maximum stress reaches yield, an expression for the elastic limit pressure is obtained as

$$C_l \cdot q_Y = C_l \cdot p_Y \cdot h = \frac{24 \cdot L}{(3 \cdot L^2 - h^2) \cdot s} \cdot \sigma_y \cdot Z_e. \tag{11a}$$

where L is the frame span and Z_e (elastic) section modulus. This simple equation can be compared with the equation in the FSICR which states that the load causing yield is (in FSICR $C_l = 1$):

$$C_l \cdot q_Y = \frac{7 \cdot m_0}{7 - 5 \cdot h / L} \frac{Z_e \cdot \sigma_y}{s \cdot L}, \tag{11b}$$

where m_0 is a boundary condition factor being between 5 and 7. These equations give very similar results and the latter one (11b) will be used in the analysis.

When the loading increases from the load causing first yield, the stress distribution at the supports include more and more plastic i.e. yield stress and two hinges are formed at the supports when the stress distribution is fully plastic. At this instant the pressure is

$$C_l \cdot q_P = C_l \cdot p_P \cdot h = \frac{24 \cdot L}{(3 \cdot L^2 - h^2) \cdot s} \cdot \sigma_y \cdot Z_P, \tag{12}$$

where Z_P is the plastic section modulus. The deformation corresponding to pressures (11) and (12) can be obtained inserting these pressures into (10).

When the stress distribution at the supports is fully plastic, the supports act as hinges. When the loading increases further, the beam behaves as a simply supported beam. Eventually the stress distribution at the mid span reaches fully plastic state. At this instant three hinges are formed and the frame collapses if the material is assumed to behave as elastic – ideally plastic. The additional load (additional to p_P) causing the three hinges is obtained setting sum of the moment of a clamped beam at the mid span due to pressure (12) and the moment caused by the additional pressure of a simply supported beam equal to $M_Y = \sigma_y \cdot Z_P$ ($M_Y =$

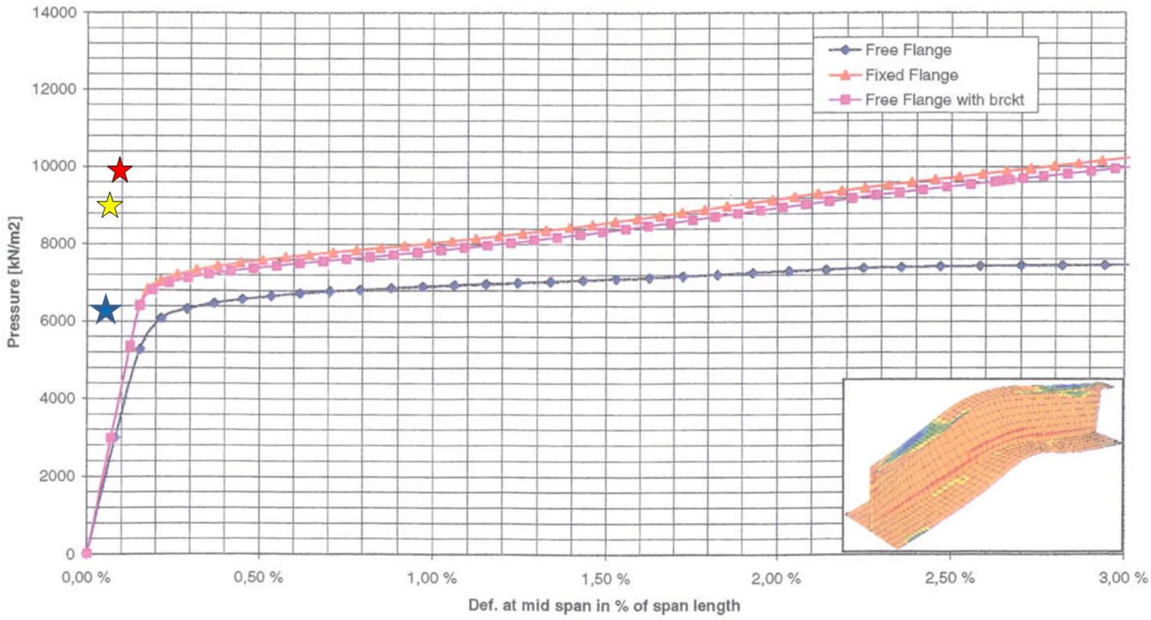
$M_C(p_P)+M_{SS}(p_{add})$ and subscript C refers to clamped and SS to simply supported). This way the ultimate pressure for a frame is obtained as

$$C_l \cdot q_U = C_l \cdot p_U \cdot h = \frac{16}{s \cdot (2 \cdot L - h)} \cdot \sigma_y \cdot Z_P \tag{13}$$

and the corresponding displacement at the mid span

$$\delta_U = \frac{12 \cdot L^5 + 18 \cdot L^4 \cdot h - 28 \cdot L^3 \cdot h^2 + 8 \cdot L \cdot h^4 - 2 \cdot h^5}{48 \cdot (2 \cdot L - h) \cdot \left(3 \cdot L^2 - h^2\right)} \cdot \frac{\sigma_y \cdot Z_P}{E \cdot I}. \tag{14}$$

The frame response formulation is simple as it does not consider the shear deformation nor the material post yield behavior. In order to check the validity of the frame plastic formulation, the results (11 – 14) are compared with results from FE calculations conducted by DNV (Holtsmark & Strömme 2004). The results of the comparison are shown in Fig. 2 (as only one frame was investigated, C_l = 1).



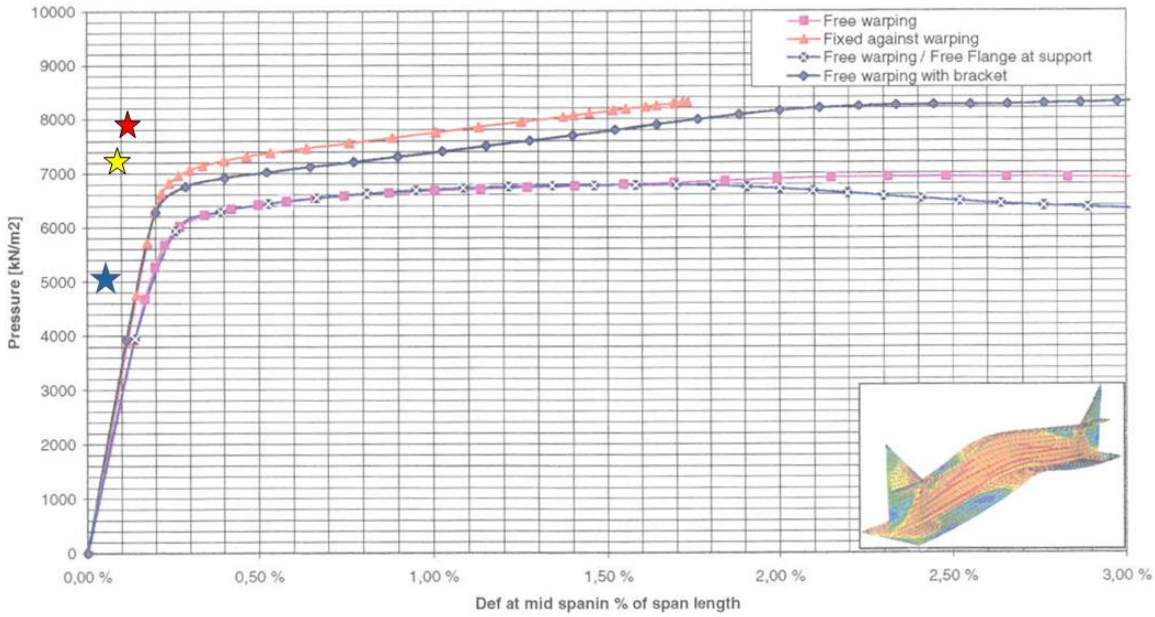


Fig. 2. Comparison of the results from the simple derivation with FE calculations (Holtsmark & Strömme 2004). The blue, yellow and red stars represent the elastic, plastic and ultimate limits, respectively. The frame span in the top figure is 2.4 m and in the lower figure 1.8 m. Other quantities are s = 40 cm, h = 40 cm, Z_e = 716 cm^3, Z_P = 1005 cm^3, I = 17310 cm^4.

The comparison shows that the displacements given by the simple formulations are smaller than those obtained by FEM. The pressure values are, however, quite reasonable. A further validation is given by the collapse equation for frames developed by Ranki (1986); this gives the ultimate pressure 10.3 MPa for the first case (L = 1.8 m) and 8.3 MPa for the second case (L = 2.4 m). These are about 5 % higher than those given by (13). Thus the present simple formulae can and will be used in this study.

2.4 Design Point

If the probability distributions of the load f(q) and F(q) and strength (s(Q) and S(Q)) are known, then the probability of failure is

$$P_f = P(Q < q) = \int_{-\infty}^{\infty}\int_{-\infty}^{q} f(q)s(Q)dqdQ = 1 - \int_{-\infty}^{\infty} F(Q)s(Q)dQ\,. \qquad (15)$$

It might be considered that the strength is not stochastic but rather described by some fixed strength value S. Then the probability of failure is, as the strength pdf is in this case the Dirac's delta function $\delta(Q-S)$,

$$P_f = 1 - F(S)\,. \qquad (16)$$

If the load and strength pdf's are assumed to be normal pdf's – this approach is called Level 2 or second moment method as normal pdf is characterized by two parameters (mean and standard deviation). The development of the structural reliability theory continues by defining the safety index β and analysing different approximate ways to determine this index – this leads, however, too far from the topic here.

Relevant here is to consider how the load pdf $f(q)$ used in (15) can be obtained and how this is related to the frequency associated with the probability of failure. Let $f_P(q)$ be the peak probability distribution function, $F_P(q)$ corresponding cumulative distribution and υ_P the peak frequency. Then the probability of failure at any peak is

$$P_{f,P} = 1 - \int_{-\infty}^{\infty} F_P(u)s(u)du,$$

where u is an integration variable. Now the probability of failure during a time period T_0 is

$$P_{f,T_0} = 1 - P(no \quad failure) = 1 - (1 - P_{f,P})^{\nu_P T_0} = 1 - \left[\int_{-\infty}^{\infty} F_P(u)s(u)du\right]^{\nu_P T_0}$$

and probability of failure during the life time is

$$P_{f,T_L} = 1 - (1 - P_{f,T_0})^{T_L/T_0} = 1 - (1 - P_{f,P})^{\nu_P T_L} = 1 - \left[\int_{-\infty}^{\infty} F_P(u)s(u)du\right]^{\nu_P T_L}.$$

After these derivations it is instructive to compare two failure probabilities; that due to all peaks during time T_0 and that due to the maxima from time period T_0. The first one is

$$P_{fT_0} = 1 - (1 - P_{fP})^{\nu_P T_0} = 1 - \left[\int_{-\infty}^{\infty} F_P(u)s(u)du\right]^{\nu_P T_0}$$

and the second

$$P_{fT_0} = 1 - \left[\int_{-\infty}^{\infty} F_{T_0}(u)s(u)du\right] = 1 - \int_{-\infty}^{\infty} [F_P(u)]^{\nu_P T_0} s(u)du.$$

These are the same only if the strength is fixed at S and thus $s(u) = \delta(u-S)$.

In the present analysis the probability of failure is not used; it is easier to use the cumulative distribution to calculate the most probable load and corresponding return period. This way insight of the quantities included in the design point description is obtained.

The concept of design point includes a definition of the limit state and the frequency this limit state is reached, assuming a certain operational spectrum for the ship. The limit state function $L(Q,q)$ includes the load q (which is determined by the operational spectrum) and the strength of the structure Q (determined by scantlings, material properties and geometry) – the latter is expressed often, as discussed above, as the load causing response up to the selected structural limit. The structural limit in the following analysis will be either stress reaching yield stress somewhere in the structure, full plasticity i.e. at some point in the structure the stress distribution is fully plastic or some specified permanent deformation –

or for frames, collapse. If the limit is denoted as *w* (being Y, P or U, respectively) and the structural details (scantlings etc.) as *d* – this can be a vector i.e. contain more than one parameter – then the limit state (which is commonly called 'strength' of the structure) can formally be presented as

$$q = f(w,d). \tag{17}$$

At the same time equation (4) gives the most probable load in a given time i.e. the return period as

$$q = g(T). \tag{18}$$

If the Gumbel I distribution is used for the load, the load versus return period is obtained by starting from eq. (4). The strength and statistics of loading can be combined to give the design equation (solving for d from $g(T) = f(w,d)$)

$$d = h(w,T). \tag{19}$$

This process of determining the scantlings is sketched in Fig. 3 where equation (17) is on the left hand side and equation (18) on the right hand side. The subject of this report is to investigate the influence of *w* and *T* on *d*. It is for example of interest to check what combinations of *w* and *T* give the same scantlings.

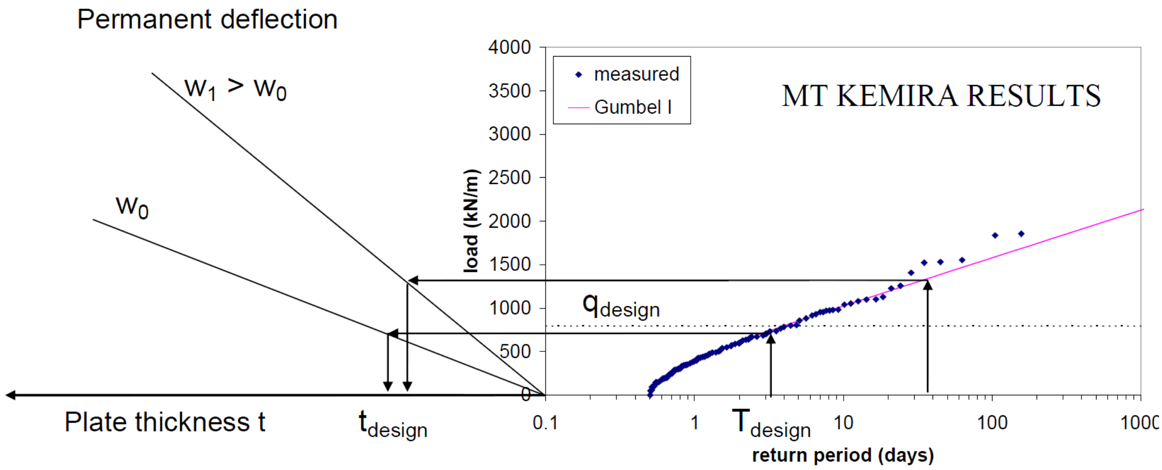


Fig. 3. A sketch of the design point definition (Riska 2007).

3. ICE LOADS

The analysis of the probability of failure or determination of the return period of the limit state requires knowledge about the statistics of ice loads. The various formulations for ice load statistics were discussed in Chapter 2.1. These considerations are now made somewhat more concrete by considering the measured data from the chemical carrier Kemira.

MT Kemira is a chemical carrier that operates between the ports Kokkola and Uusikaupunki in Finland and central European ports. She visits Finland often with a rotation of about one week. Her ice class is IA Super with a length of L_{pp} = 105,0 m, B = 17.5 m, power 3400 kW and deadweight 5800 dwt. This ship was selected for measurements as she navigates regularly in the most ice condition in the Baltic – thus her operational spectrum can be assumed to represent a typical Baltic merchant ship, of her size and shape, naturally.

MT Kemira was instrumented to measure the ice load on one frame (labelled FFR in Fig. 4) as well as the plate and frame stress response (labelled PL and FN). The frame load measurement was carried out by using shear stress gauges attached at the neutral axis of the frame; the difference of two shear stresses on the same frame is proportional to the load on the frame between these gauges. The ship and measurement locations are shown in Fig. 4.

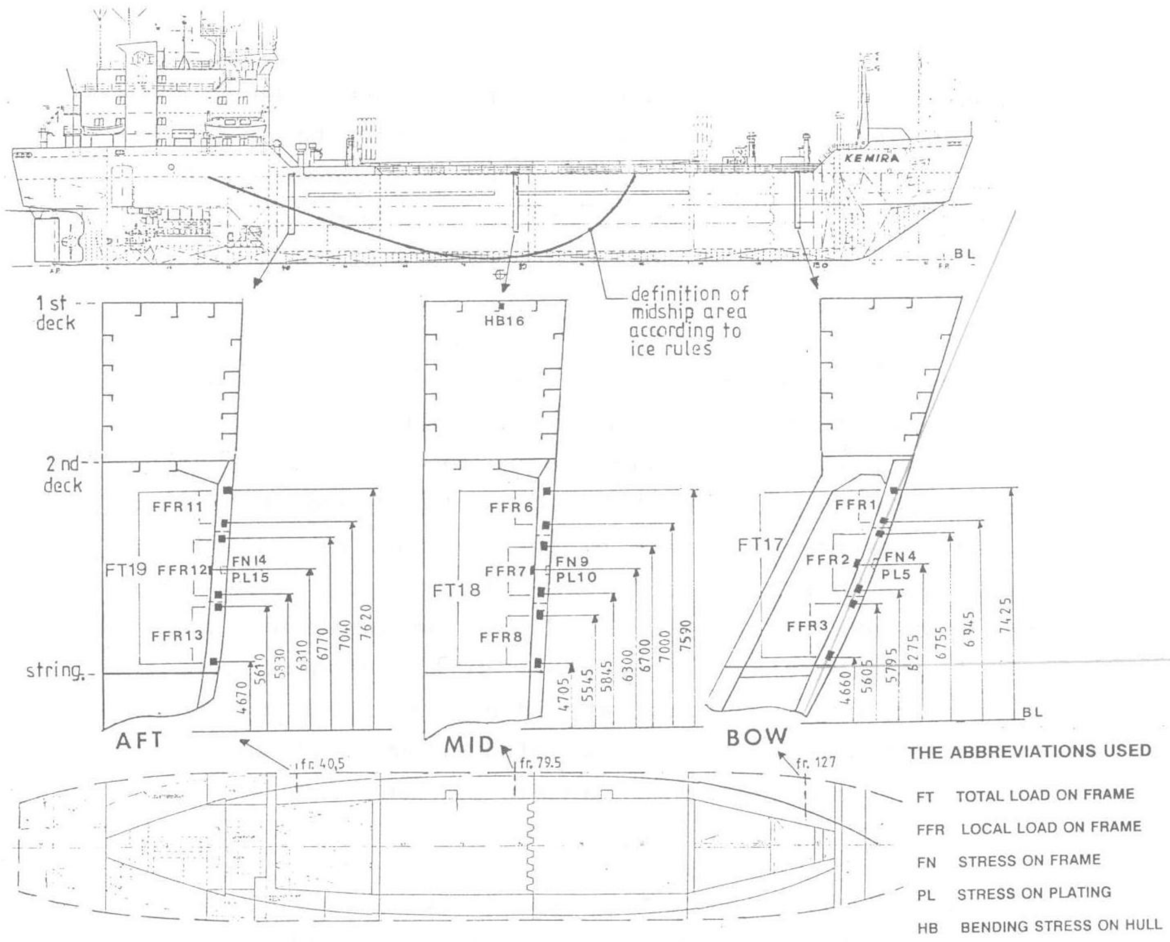


Fig. 4. The ice load measurement system installed onboard MT Kemira (Kujala 1989).

The data used from MT Kemira include the frame loads denoted by *F* [kN]; these were measured at the bow, midship and stern areas of the ship hull, and at two or three draughts. The stern load measurement location is actually at the stern shoulder i.e. at the boundary between the midship and stern hull area. The data measured and also the measuring system is described in several reports, see Kujala (1989), Gyldén & Riska (1989), Muhonen (1991 and 1992).

The measurement system does not record the time histories of the signals but calculates instead two histograms for each measurement channel. One histogram is the digitized samples from each measurement channel with a sampling rate of 100 Hz. The other histogram is the peak histogram where each peak is calculated using the Rayleigh separation for the peaks (with 25 % of the peak value as the separator). These histograms were stored after each 12 h period during the whole winter in years 1985 – 1991.

The data used here are derived from the maxima of each 12 h peak distributions. Only the maxima when the ship has been sailing in ice are included. Before using the data some processing is to be done. The first assumption made is to omit the smallest peaks as these contain most likely some electronic noise as well as ice induced response. Specifically, peaks smaller than 20 kN are neglected. The other assumption made is to collate the data so that from each 12 h period, the maximum frame load of the measured peaks at the same frame is taken to the data. Thus the data consist of MAX[FFR1,FFR2], MAX[FFR6,FFR7] and MAX[FFR11,FFR12] where FFR stands for frame load in the gauge denotation used in the measurements.

The data used are in a form of histograms for bow, midship and stern frame loads. The measured load is converted into a line load by dividing the measured force F by the frame spacing ($q = F/s$, s = 35 cm), thus the basic data is in form of $\{q_i\}$, i=1,...,N where each q_i is from some 12 h period during years 1985 – 91 and N is the number of 12 h periods included in the data. In order to use the data in the subsequent analysis, Gumbel I distribution is fitted to the data. The data and the fits are presented in the table 2. The data and the Gumbel fits are shown in Fig. 5a...f. The histograms were calculated using a bin width of 100 kN/m and the plots versus the return period were made plotting q versus $T = \frac{1}{\nu_0}\frac{1}{1-G(q)}$ where ν_0 = 1/(0.5 day) = 2/day.

Table 2. Data about frame ice loads

Quantity	Bow	Midship	Stern
No. of points N	304	309	388
Mean [kN/m]	460.2	241.3	244.6
STD [kN/m]	334.6	178.4	203.8
Gumbel para. c [m/kN]	0.00383	0.00719	0.00629
Gumbel para. u [kN/m]	309.7	161.0	152.9

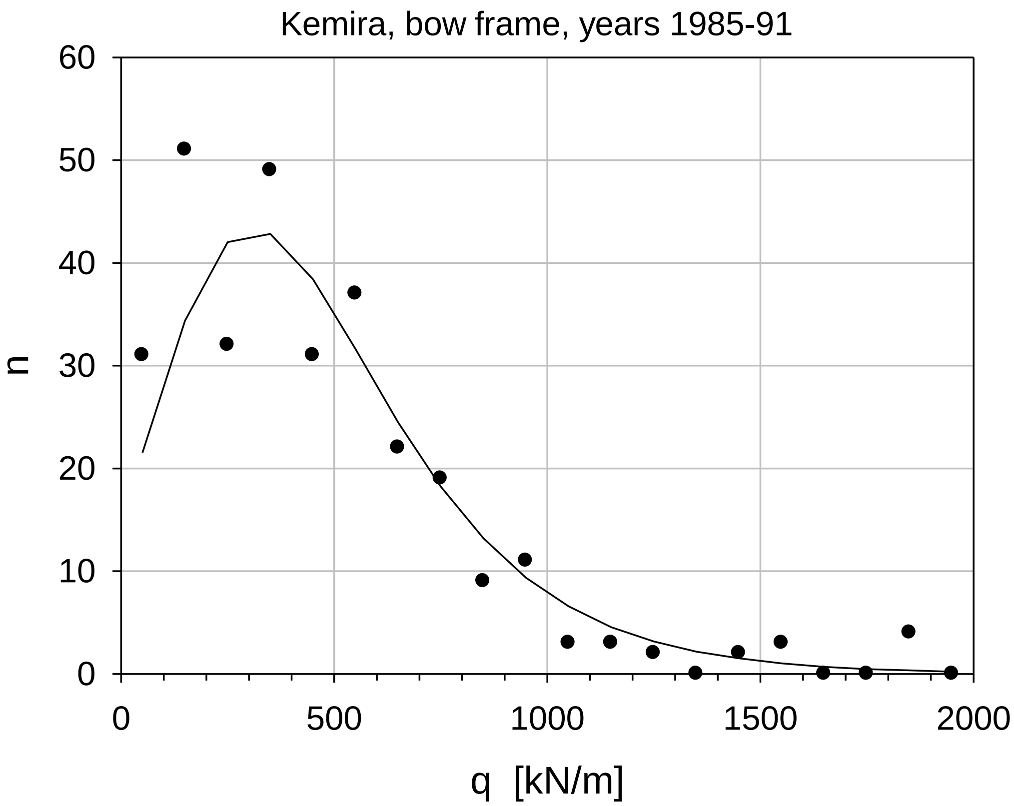


Fig. 5a. The measured load histogram and fitted Gumbel I pdf for bow frame loads.

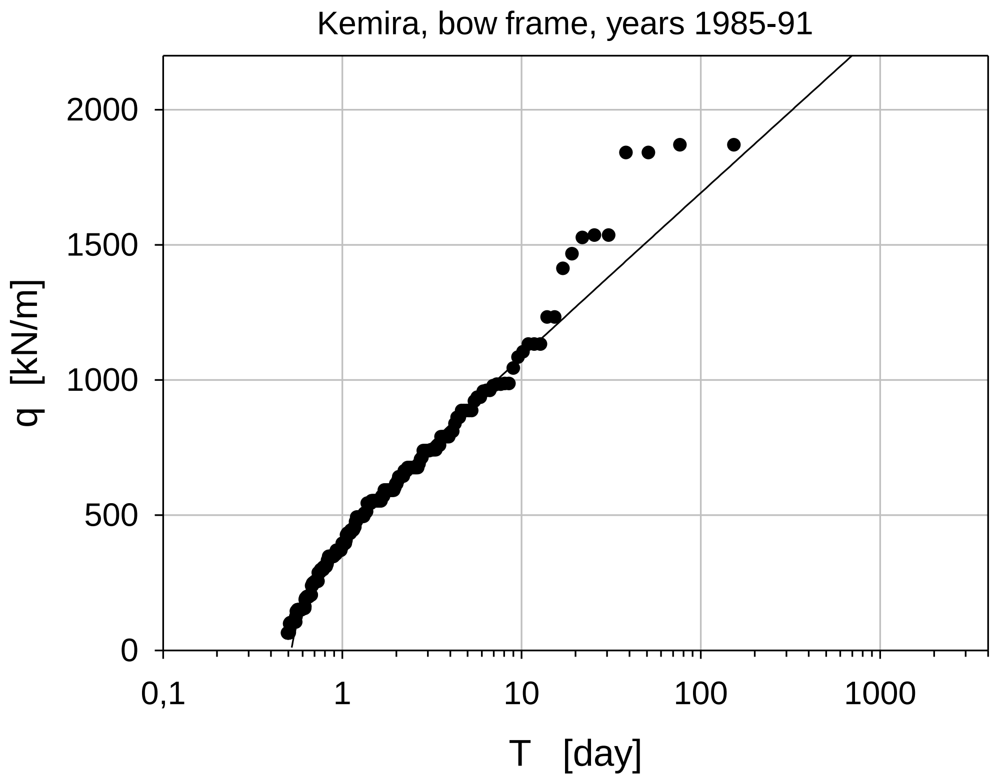


Fig. 5b. The measured and estimated (according to Gumbel I pdf) loads versus the return period for bow frame loads.

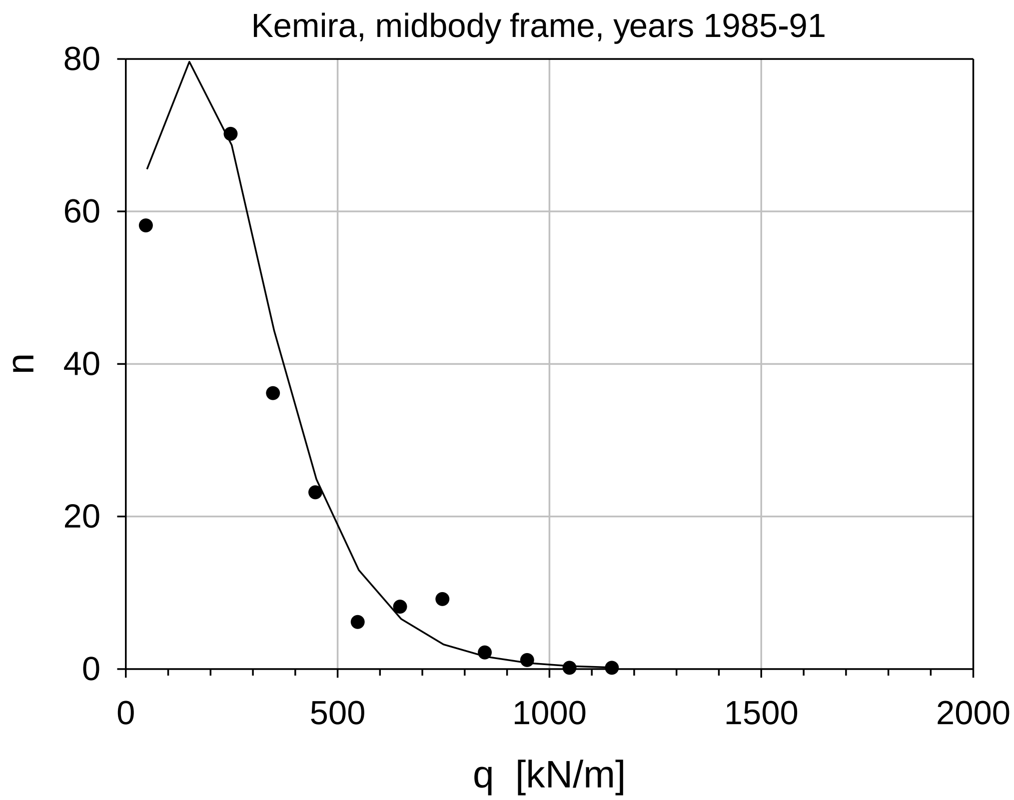


Fig. 5c. The measured load histogram and fitted Gumbel I pdf for midship frame loads.

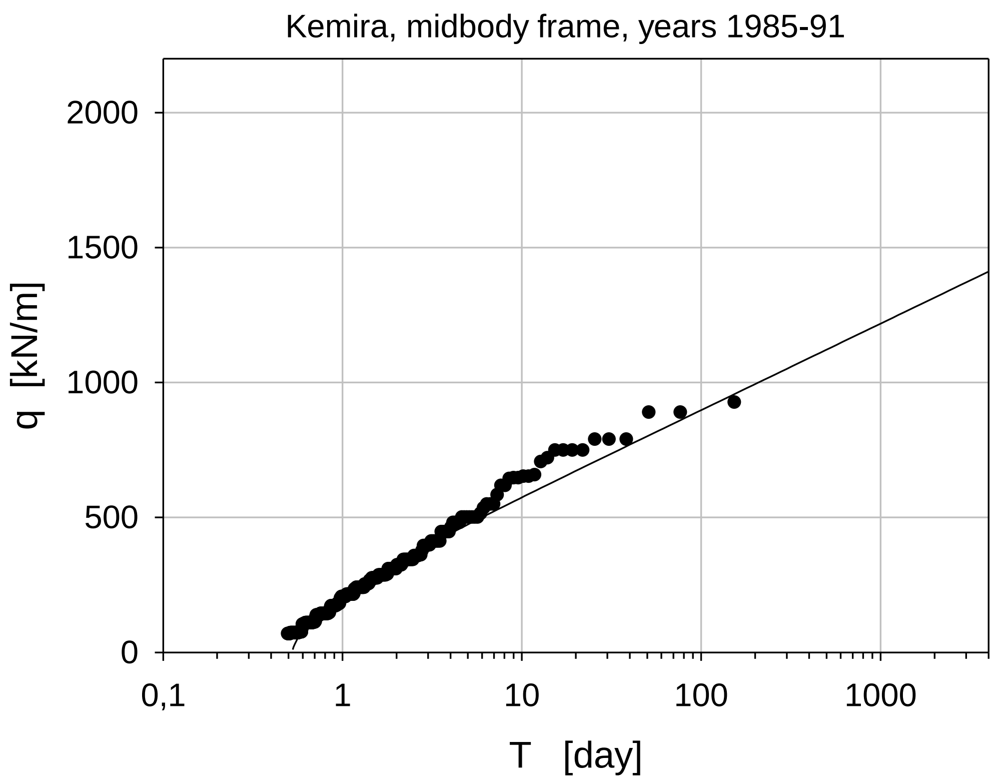


Fig. 5d. The measured and estimated (according to Gumbel I pdf) loads versus the return period for midship frame loads.

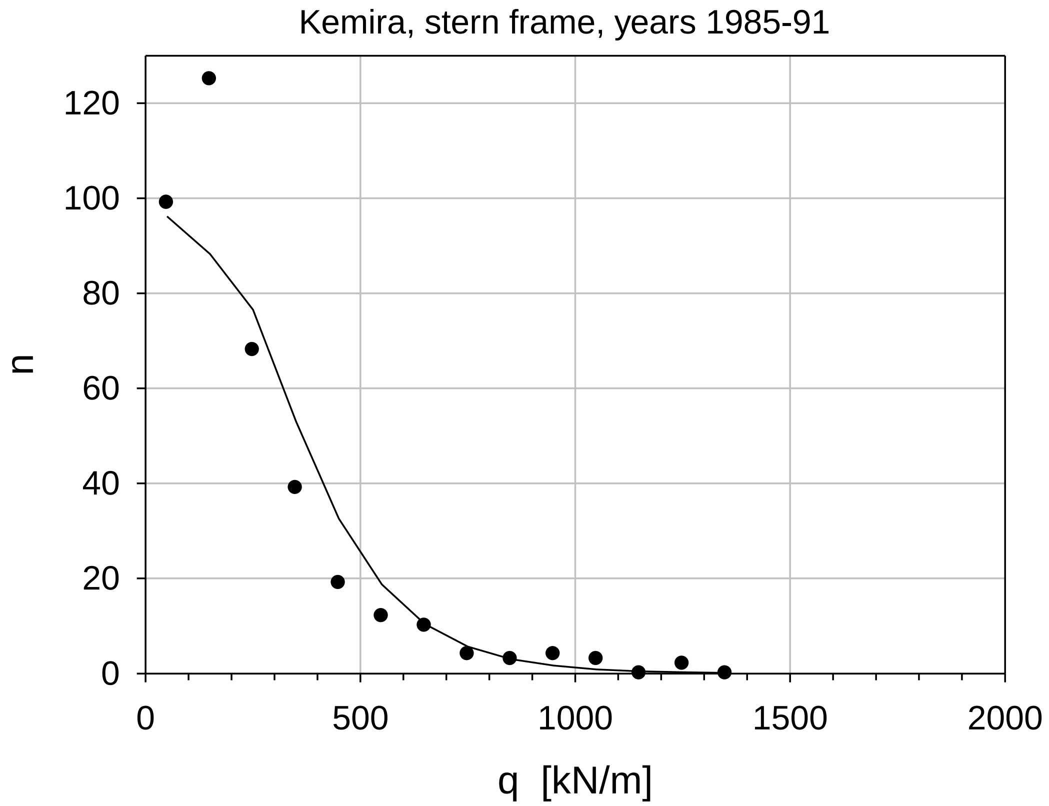


Fig. 5e. The measured load histogram and fitted Gumbel I pdf for stern frame loads.

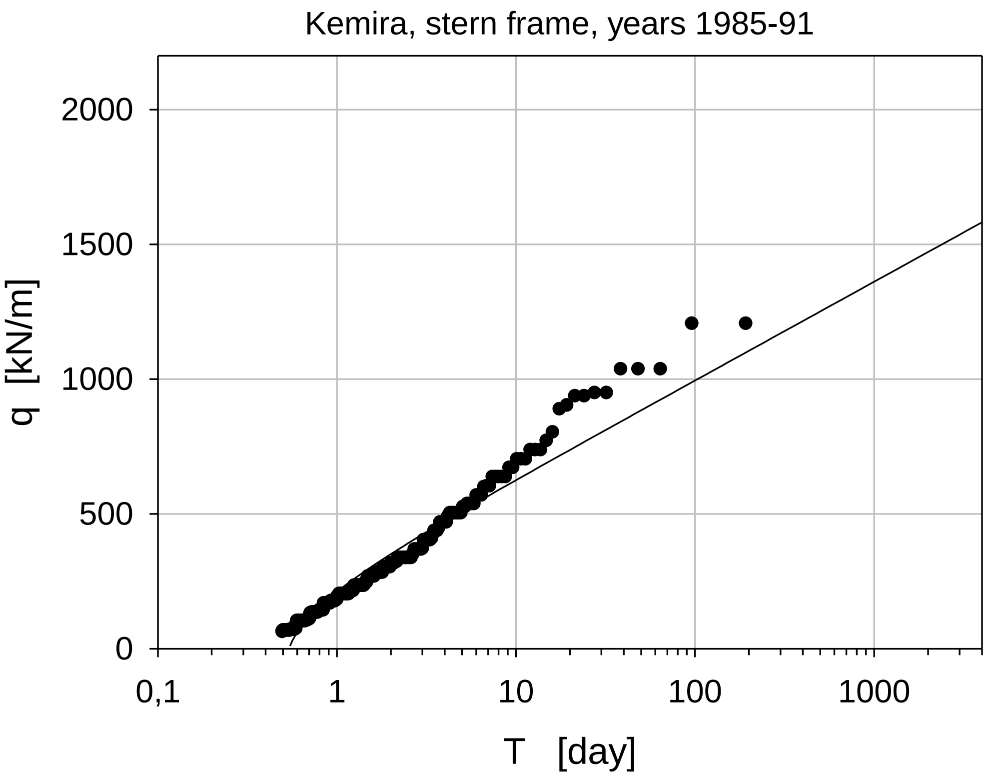


Fig. 5f. The measured and estimated (according to Gumbel I pdf) loads versus the return period for stern frame.

The advantage of using the Kemira data from these seven years is that it integrates different winters (three severe, two average and two mild) as well as the ice thickness development during each winter. The disadvantages include the dependency on MT Kemira size, hull shape and navigation spectrum (MT Kemira visited mostly the ports in Uusikaupunki and Kokkola in Finland). Also the fact that during each 12 h period the ship did not navigate in ice the whole time – in fact the ice navigation was less than 12 h for most of the 12 h periods (see Muhonen 1992, Appendix 3).

In order to have some insight on the data, the statistical distributions are investigated shortly. The basic distribution is the distribution of peaks, $f_P(q)$. The bow frame load data from 16.3.1991 am is used for this demonstration, data from Muhonen (1992). The ship was operating this day through the Quark having left Kokkola at midnight. The maximum level ice thickness during this time and this area was about 50 cm. These data and the Gumbel I distribution fitted to the peaks are shown in Fig. 6 (the Gumbel I parameters are c = 0.0152 m/kN and u = 85.6 kN/m). It is quite clear that the Gumbel distribution underestimates the number of larger peaks. This becomes clear if the Gumbel distribution of the peaks is extrapolated to the whole measurement period (the ship was in ice during this period in total T = 6.5 h and in total N = 2040 peaks were measured). The extrapolation gives for the most probable maximum during this time

$$F_T(q) = [F_P(q)]^N = EXP[-EXP[-0.0152m/kN(q-585.8kN/m)]] .$$

The two corresponding pdf’s calculated from $F_P(q)$ are compared in Fig. 7. It is evident that the Gumbel I distribution underestimates somewhat the maximum value – some other distribution like Weibull distribution might give a better fit on the peak distribution. This warrants a study in itself.

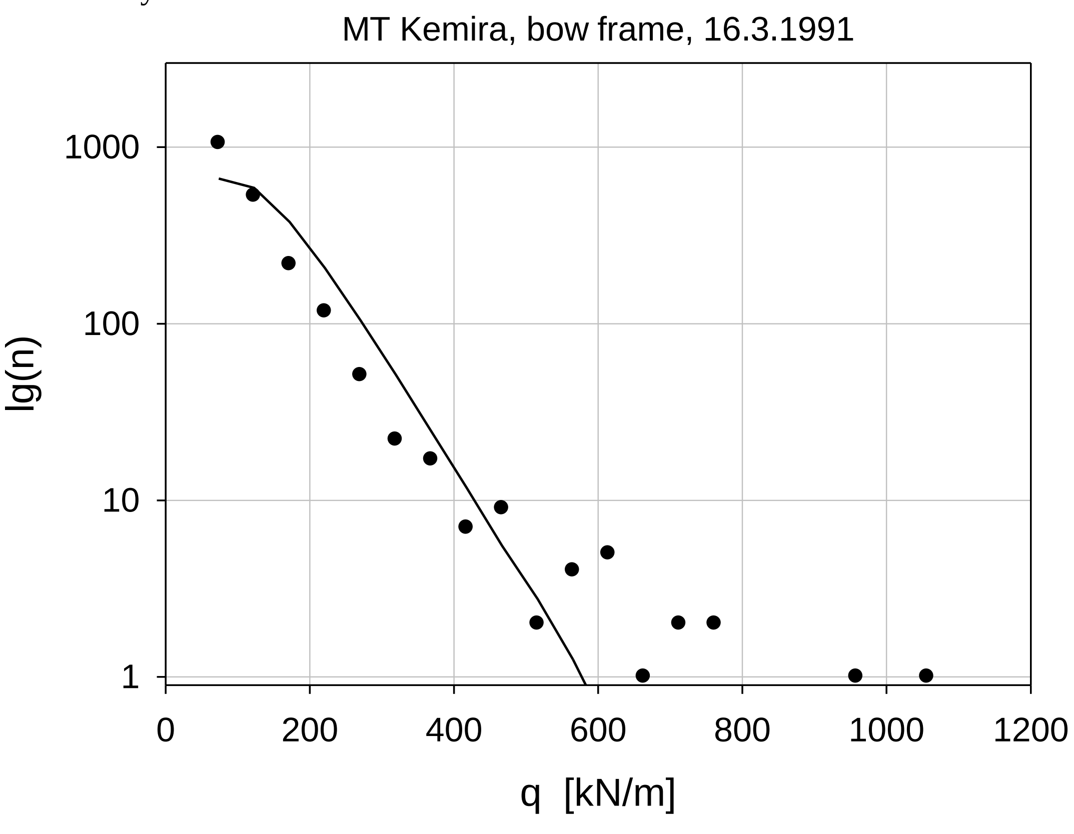


Fig. 6. The measured histogram of peaks in 16.3.1991 (am) and the fitted Gumbel I distribution.

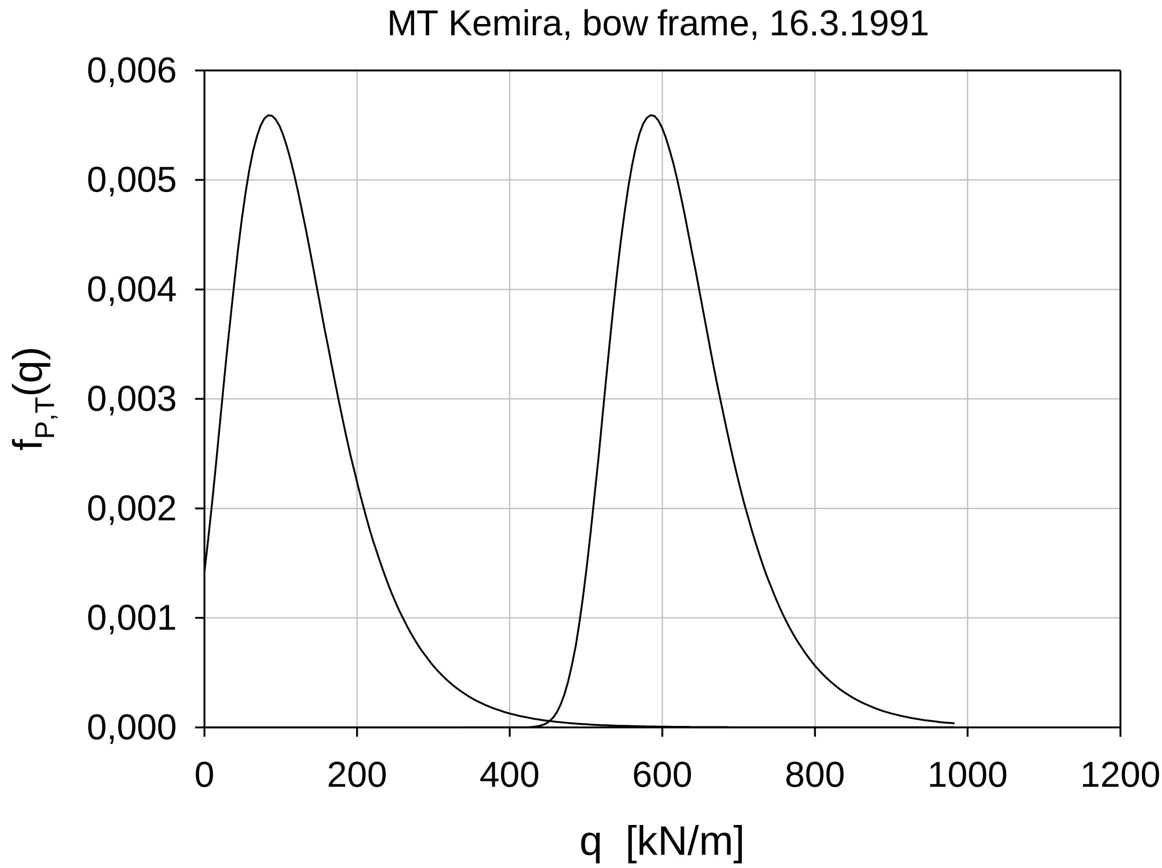


Fig. 7. The fitted peak pdf and the extrapolated pdf for the 6.5 h maximum value.

It is also of interest to compare the distribution fitted to measured 12 h period maxima with the extrapolated distribution derived from the measured peak histogram from 16.3. The comparison is shown in Fig. 8. The fit to 12 h maxima gives a far wider distribution which is clear as all kinds of ice conditions are included in the data whereas the extrapolated peak distribution is quite sharp with a most probable value of about 630 kN/m. Also the ice conditions during 16.3. correspond to the most severe annual conditions.

This short analysis of the maxima raised some questions concerning the extrapolation of loads and also about the connection of the estimated pdf's to ice conditions. It would be interesting to study the relationships between the peak distribution, distribution of maxima from certain time periods and the method of extrapolation to lifetime values. This falls, however, outside the present study where the collections of 12 h maxima are used as the basis for analysis of the design point.

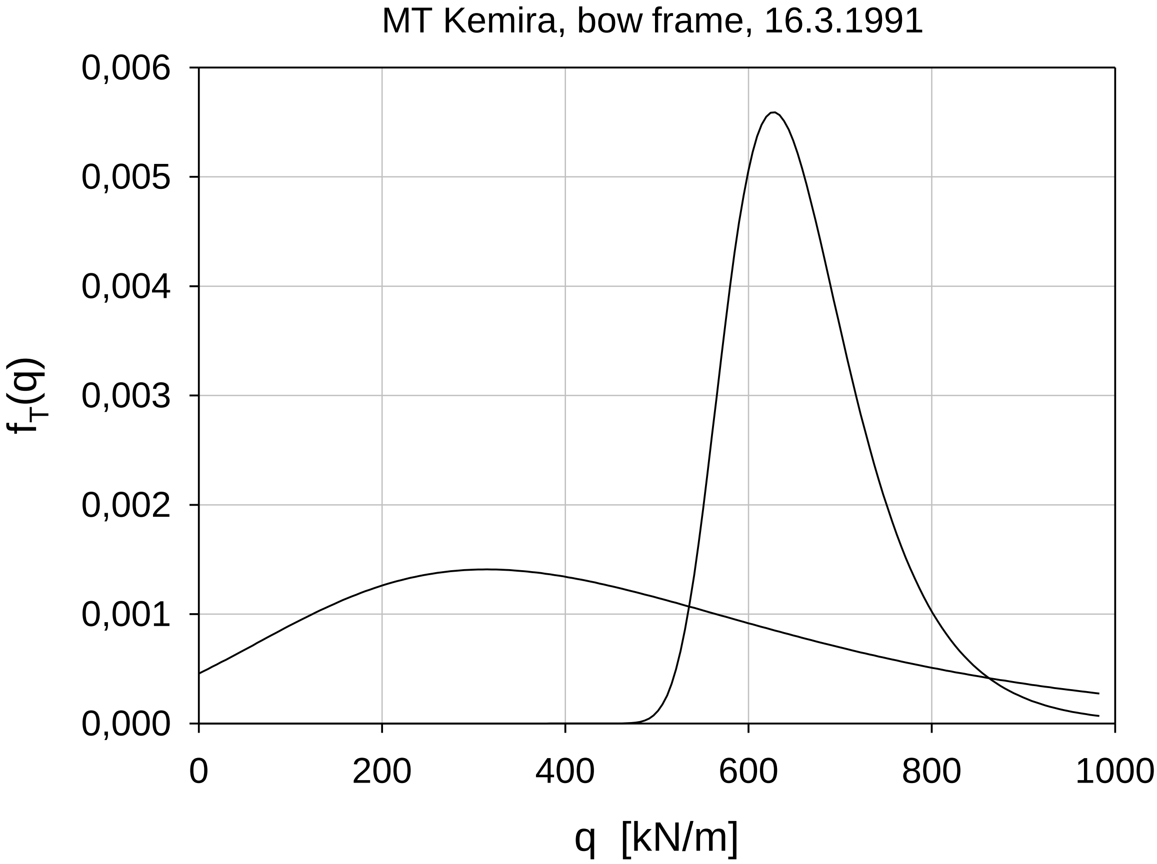


Fig. 8. The extrapolated peak histogram to 12 h period (right pdf) compared with the distribution of the maximum for 12 h period from years 1985-91 (left pdf).

4. THE DESIGN POINT

The design point was defined in Chapter 2.3 to be the relationship between the scantlings (denoted as *d*), limit state used (denoted *w*) and the frequency of loading i.e. frequency of reaching the limit state – equation (19) formally describes this relationship. Three different limit states are considered for framing: yield, full plasticity at the supports (2-hinge formation) and collapse (3-hinge formation). For plating the yield, full plasticity (characterized by $w_P = 0$) and some permanent deformation are considered as the cases to be analyzed.

The equation (19) can in principle be obtained combining the load formulation given in Chapter 3 and the limit state formulations given in Chapter 2.2. The resulting equations would not, in themselves, give much insight; thus the whole analysis is made using MT Kemira as an example. This way the values of most of the scantlings like the frame spacing, frame span, frame boundary conditions and the yield strength of the steel can be fixed. Thus the frame spacing used in the subsequent analysis is s = 35 cm, frame span L = 3.5 m and yield strength σ_y = 235 MPa.

Three different hull areas are investigated; bow, midship and stern, using the measured loading as representative for these areas. This is not strictly correct as the bow as well as stern area contains a range of hull angles. The stern measurement position is where the frame angle is almost vertical – thus these loads can be considered as conservative what comes to the influence of hull shape. The bow load measurement position is where the waterline angle is $\alpha = 19^o$ and the frame angle is about $\beta = 24^o$. This position is at about the centre of the bow area length and thus the measured loads can be considered as characteristic for the whole Kemira bow, on average.

The ice class of MT Kemira is IA Super. The calculated scantlings are compared with the minimum values required by the Finnish-Swedish Ice Class Rules (FSICR, Trafi 2010) and also with the ice class PC6 of International Association of Classification Societies Polar Class Rules (IACS 2007). These two classes are considered generally as approximately equivalent – this equivalency is one of the motives of this study as the design point in the Finnish-Swedish Ice Class Rules includes the yield (Y) limit state and the IACS rules a plastic limit state (either P or U, not clear which). The required scantlings of MT Kemira for these two classes are given in Table 3. It should be noted that the required section modulus for the ice class PC6 is plastic section modulus while for ice class IA Super it is elastic. The plate thickness given is the net thickness.

Table 3. The required scantlings for MT Kemira (both the elastic and plastic section modulus are given, Z_e/Z_p)

Hull area	Scantling	IA Super	PC6	Kemira, as built
Bow	Z [cm^3]	713	3100	773 / 1010
	t [mm]	17.4	18.0	21
Midship	Z [cm^3]	412	1230	390 / 553
	t [mm]	13.9	11.8	16
Stern	Z [cm^3]	309	1094	384 / 490
	t [mm]	12.1	11.1	16

Some comments on these calculated scantling values are in place as slightly different values have been shown in Hänninen (2002) and Kaldasaun (2010). In case of the FSICR the slight difference comes from the boundary condition factor m_0 and the frame span (as in principle the whole span without reduction of brackets is to be used in FSICR). A larger difference results from IACS rules for frame section modulus. The sensitivity of the required section modulus to the quantity $k_z = z_p / Z_P$ where z_p is the sum of local plastic section moduli of the associated plating and the frame flange explain the differences (and it is not exactly clear how to calculate the local plastic section modulus for bulb-profiles).

Finally, a note on the quantities describing the loading. The measured loading used in the analysis is the line load q which is the same as the average ice pressure p multiplied by the load height i.e. $q = p \cdot h$. The load height or load lengths are not given by the measurements and this causes some uncertainty in the analysis, uncertainty which is tackled below. For frames the load height does not have much influence on the response, especially as the frame span used is long compared to the assumed load height. Plating response is assumed to be quite insensitive to the load length.

4.1 Load Height

The load height h and the coefficient to describe the pressure distribution C_f are used to calibrate the plating response equations (5,...,8). For the calibration the plating response measured from MT Kemira as well as the frame load are used. The calibration principle is to determine the stress and load at certain return period and then use eq. (5) to set the C_f (or h). The return period is determined using the return period for exceeding the nominal yield stress in plating.

Altogether 367 12 h plate stress maxima at the bow frame were recorded during the Kemira measurement campaign (see Muhonen 1992). The corresponding Gumbel I parameters of the plate stress pdf are c = 0.01798 1/MPa and u = 54.12 MPa. At the same time there were 18 instances when the 12 h period maximum plate stress exceeded 235 MPa – this corresponds to a return period of 10.2 days. Using the Gumbel I fit given in Table 2, this corresponds to a line load of 1090 kN/m. Similar calculation using the fitted stress distribution gives a stress of 220.4 MPa.

Using these load and response values; and MT Kemira as built scantlings, the equation (5) can be plotted. The plot is given in Fig. 9. This plot suggests that at the nominal load height for ice class IA Super ships, h = 0.35 m, the coefficient value must be C_f = 0.75. This is exactly the value used in the ice rules. If, on the other hand, the value of the flexibility coefficient is assumed to be one, the load height would have to be about 58 cm. As this load height is considered somewhat high, the analysis is completed using the nominal load height of 35 cm and C_f = 0.75.

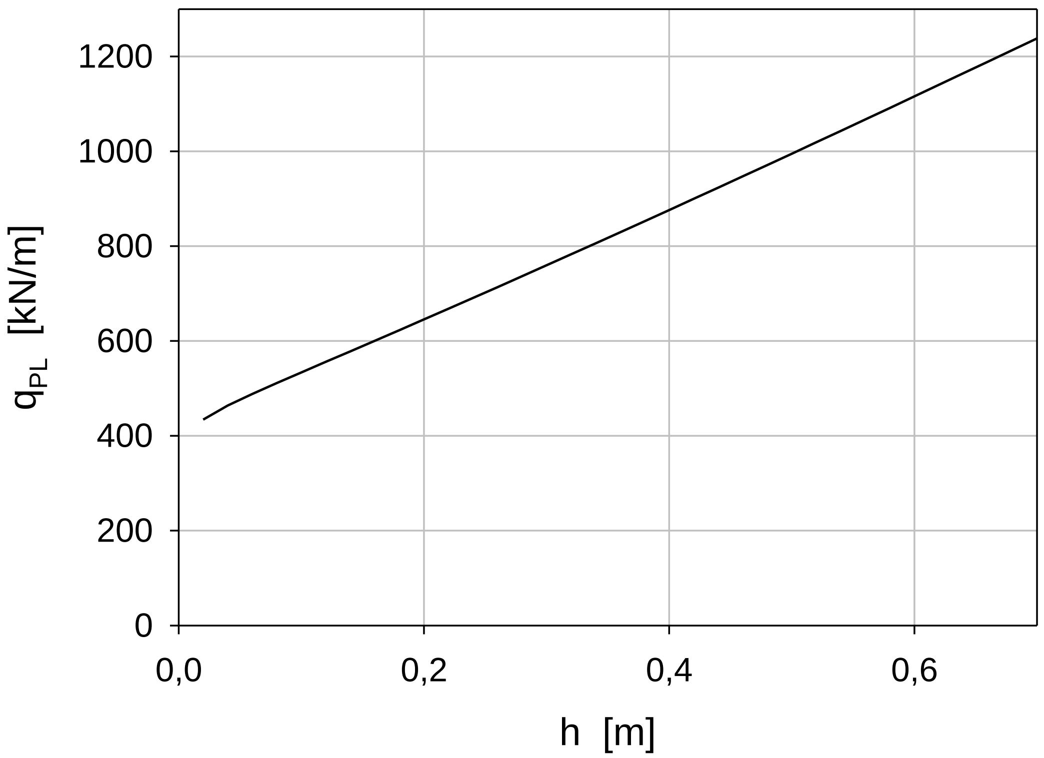


Fig. 9. The plot of the needed equivalent plate force to create the measured response, plotted versus the load height.

4.2 Plating Analysis

When the load statistics and strength equation have been derived, the required scantlings can be determined versus the return period using the limit state as a parameter. These plots for plate net thickness are shown in Figs. 10 – using MT Kemira as an example. In these plots also the required net plate thickness according to ice class IA Super and PC6 are shown.

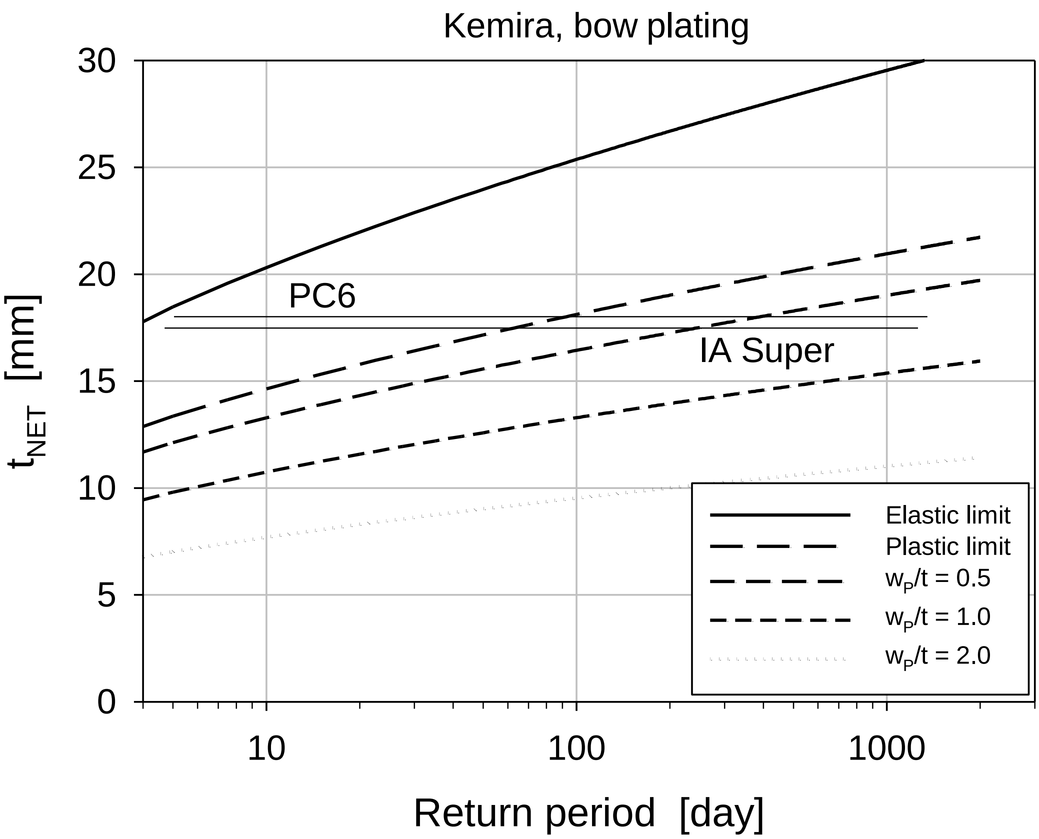


Fig. 10a. The plate thickness corresponding to different limit states plotted versus return period, bow area.

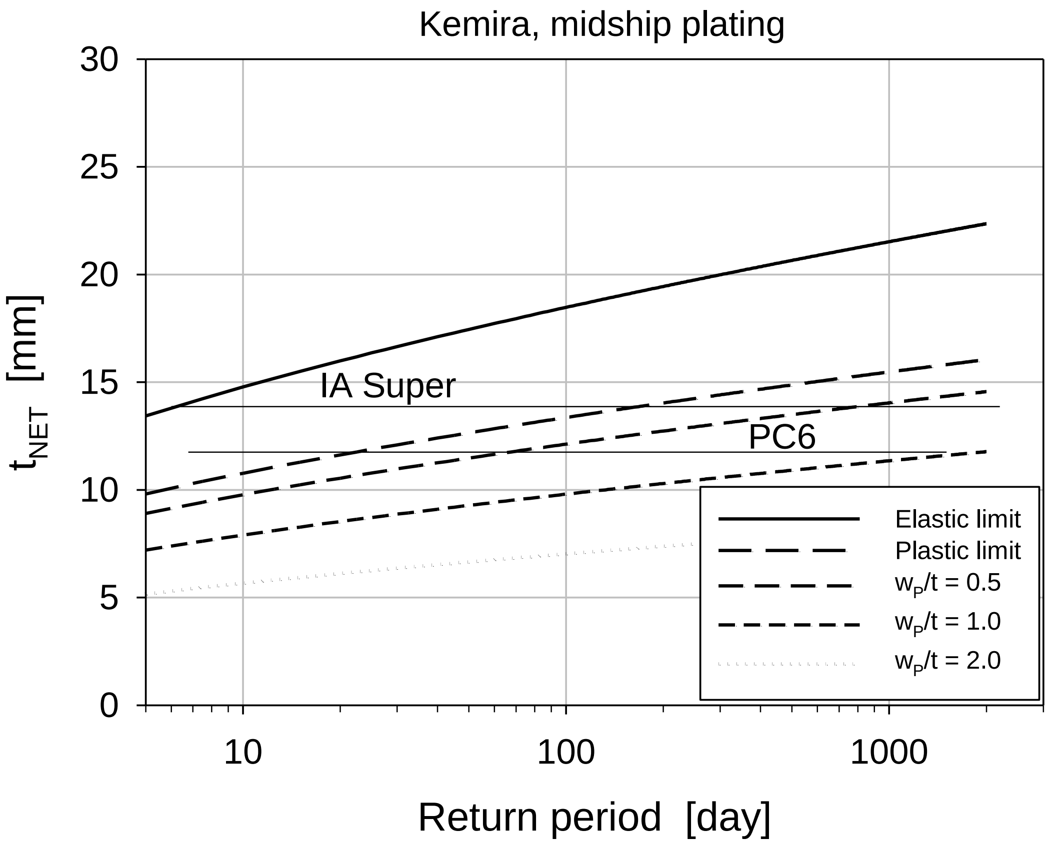


Fig. 10b. The plate thickness corresponding to different limit states plotted versus return period, midship area.

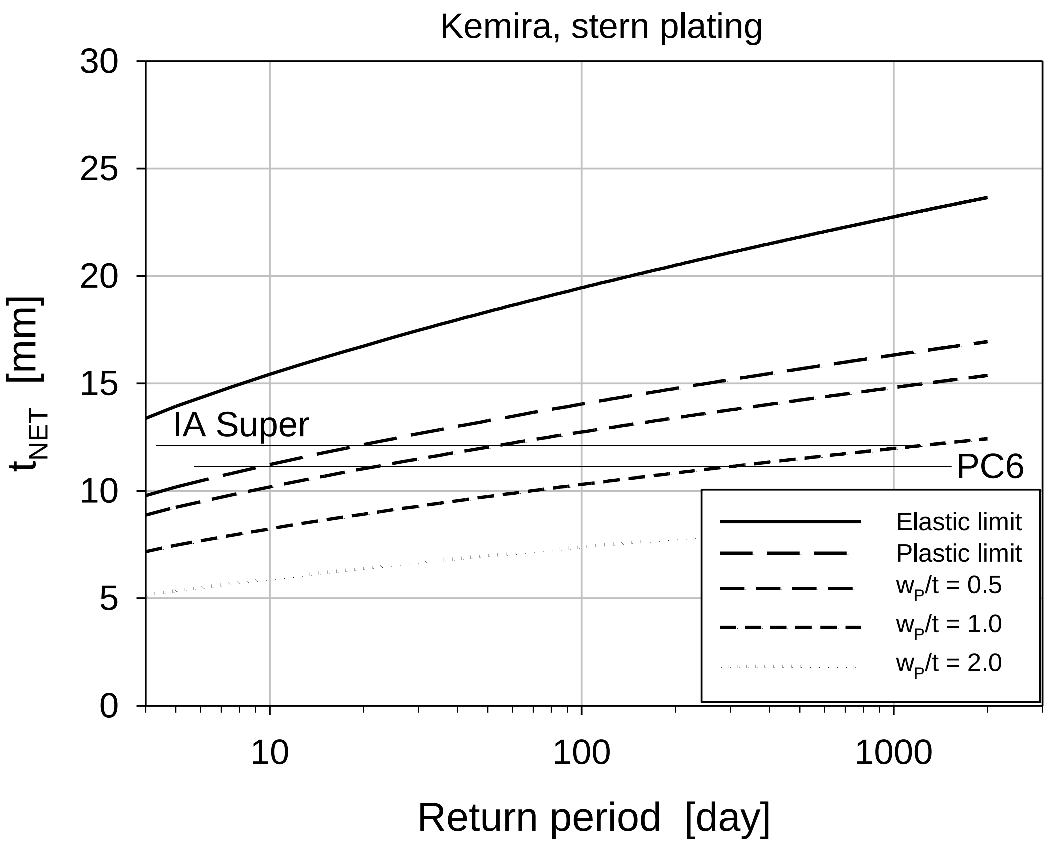


Fig. 10c. The plate thickness corresponding to different limit states plotted versus return period, stern area.

The plots in Fig. 9 can be interpreted in several ways. One way is to check what is the most probable maximum response of MT Kemira during her lifetime or during one winter. This can be estimated assuming 50 days in ice per winter (this is the average from the measurement period). This means that in her lifetime the ship would be 1250 days in ice if she would be in the same traffic throughout. The other way to look at the results is to estimate the return period of each limit state using the MT Kemira actual scantlings given in Table 3. These results are shown in Table 4 below. Partial plasticity refers to a stress distribution where the outer fibres have yielded but closer to the neutral axis the stress is still below yield.

Table 4 suggests that the design of the different hull areas is quite balanced as the return periods of various limit states is quite similar, these return period suggest further that the midship is slightly stronger than the bow and stern – this is also showed by the measured plate stresses (Muhonen 1992) even if the difference in measurements is slightly more pronounced (there was 18 cases when the stress was larger than 235 MPa at bow as compared with 3 cases for midship and stern). The results suggest further that some denting is to be expected during the ship life time. The situation changes somewhat if, instead of as built plate thickness, the required plate thickness in ice class IA Super is used. The stern area becomes much weaker.

It should be noted that these results have been obtained using the nominal yield strength of 235 MPa. For this kind of steel the average yield strength of a large sample of steel specimens is about 300 MPa (Kaldasaun 2010). As the strength of plating is proportional to the square root of the yield limit, using the average instead of nominal yield limit would increase the strength by 13 %. This effect is analyzed in the following discussion.

Table 4. The results from the design point study of the plating (the arrows show the parameter used to determine the return period/limit state).

	Limit state	Return period As built scantlings	Return period Required (IA Super) net scantlings
Bow	$w_P/t \approx 0.2$	← Life time	-
	Partial plasticity	← Winter	-
	Yield (Y) →	10 days	3.5 days
	Plasticity (P) →	21.4 years	64 days
	$w_P/t = 0.5$ →	> life time	4.8 years
	$w_P/t = 1.0$ →	> life time	> life time
Midship	Plastic limit	← Life time	-
	Partial plasticity	← Winter	-
	Yield (Y) →	21 days	6.5 days
	Plasticity (P) →	life time	3.3 years
	$w_P/t = 0.5$ →	> life time	15.6 years
	$w_P/t = 1.0$ →	> life time	> life time
Stern	$w_P/t \approx 0.2$	← Life time	-
	Partial plasticity	← Winter	-
	Yield (Y) →	13.5 days	3.5 days
	Plasticity (P) →	13.2 years	19 days
	$w_P/t = 0.5$ →	> life time	55 days
	$w_P/t = 1.0$ →	> life time	22 years

4.3 Load Length

The coefficient C_l is used calibrate the frame response equations (11, 12 and 13). For the calibration the frame stress response measured from MT Kemira as well as the frame load are used. The calibration principle is to determine the stress and load at certain return period and then use eq. (11b) to set the value of C_l. The return period is determined using the return period for exceeding the nominal yield stress in framing.

Altogether 298 12 h frame stress maxima at the bow frame were recorded during the Kemira measurement campaign (see Muhonen 1992). The corresponding Gumbel I parameters of the plate stress pdf are c = 0.02490 1/MPa and u = 45.52 MPa. At the same time there were 4 instances when the 12 h period maximum plate stress exceeded 235 MPa – this corresponds to a return period of 37.2 days. Using the Gumbel I fit given in Table 2, this corresponds to a line load of 1433 kN/m. Similar fit to the stress data gives a stress of 218.4 MPa.

Using these load and response values; and MT Kemira as built scantlings, the equation (11b) can be used to calculate the coefficient C_l. The value obtained is $C_l = 0.62$. It should be noted that the reference load in this is the frame load $F = p \cdot h \cdot s$. If the reference load would have been $F = p \cdot h \cdot l$, then an idea of the form of the coefficient C_l would have been required. Some validation for the coefficient can be obtained from the studies about strength of frames. When the case where only one frame is loaded (Varsta et al. 1978) many frames structure is compared to the case where the load length is long covering several frames (e.g. Ranki 1986), the ratio of the ultimate strengths is about 0.53 (Kaldasaun 2010). The value derived here corresponds to the yield limit, thus the value 0.62 can be considered passable and will be used in this analysis.

4.4 Frames

The plots of the elastic section modulus versus the return period, using the limit state as a parameter, are shown in Figs. 11. These plots are made using the geometry of the shell structure of MT Kemira. The section modulus shown is the elastic section modulus – the plastic section modulus obtained from the plastic analysis is converted to the elastic one using the ratio of plastic to elastic section modulus of MT Kemira frames ($Z_P/Z_e = 1.31$ for bow frames, 1.43 and 1.28 for the midship and stern frames, respectively).

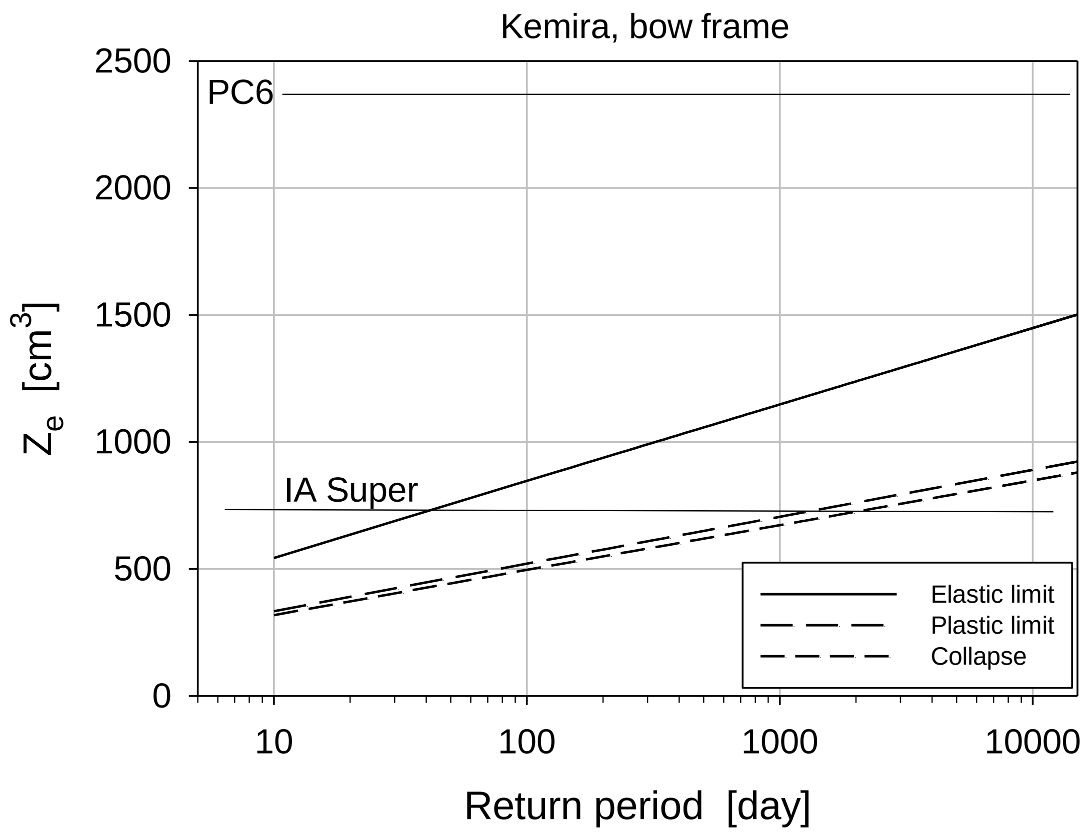


Fig. 11a. The frame section modulus corresponding to different limit states plotted versus return period, bow area.

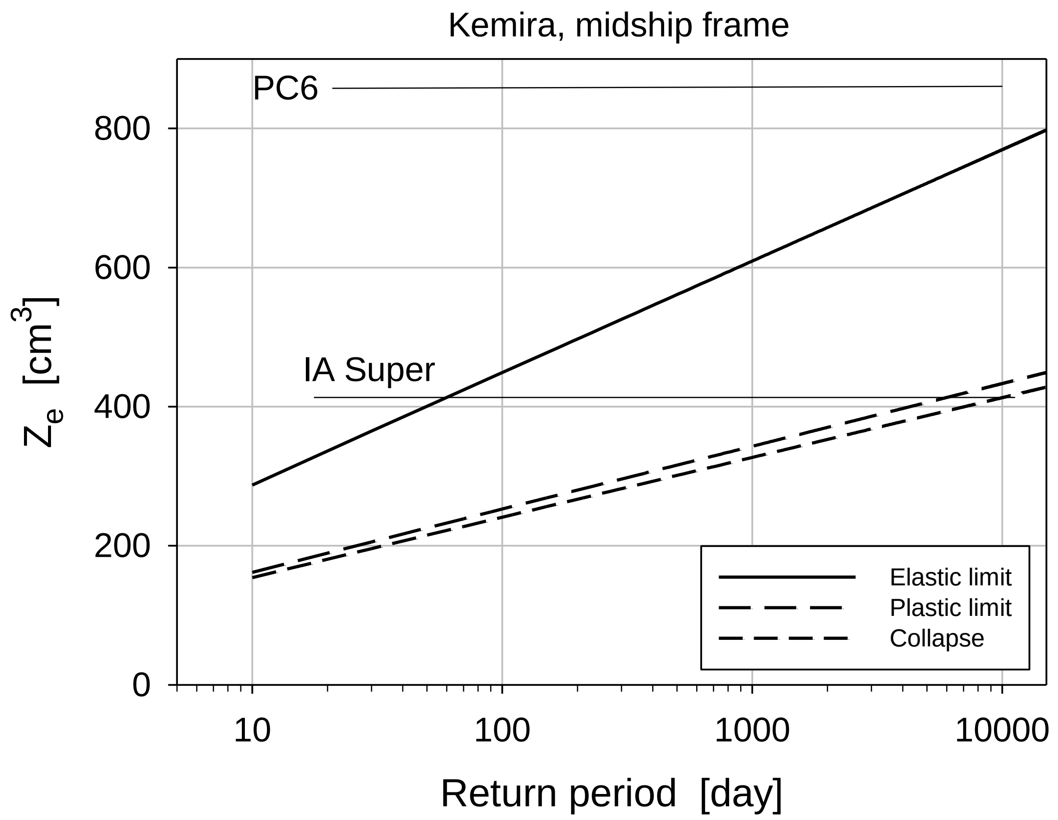


Fig. 11b. The frame section modulus corresponding to different limit states plotted versus return period, midship area.

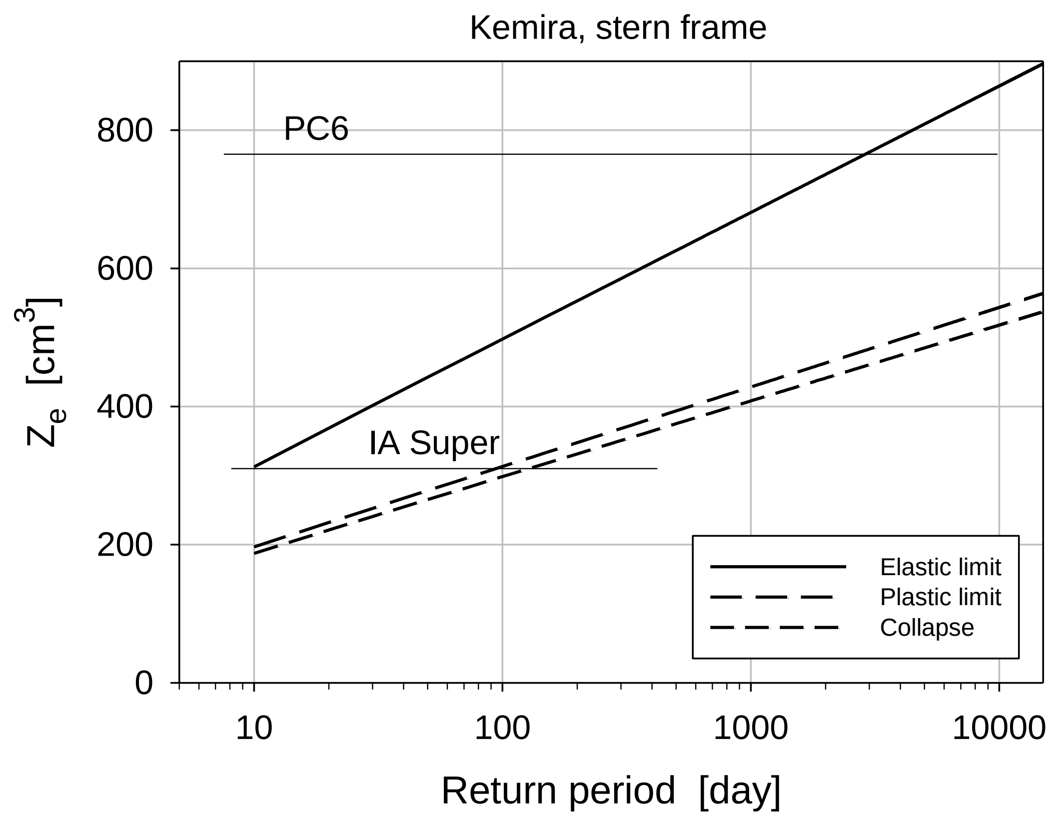


Fig. 11a. The frame section corresponding to different limit states plotted versus return period, stern area.

As was the case for plating, also these plots can be interpreted in various ways. Here the return period of various limit states is estimated; the results are given in Table 5. These results show directly that the return periods are very short, much shorter than the measurement results suggest. In relative terms the results show that the bow, midship and stern areas, as built, are quite close in strength. If the comparison is made using the required scantlings instead of the as built scantlings, the stern appears much weaker than the bow or midship. One of the possible reasons for the difference between the observations is the use of nominal yield strength instead of the actual (or average) one. As the strength of frames is directly proportional to yield limit, this effect would make the frames about 28 % stronger at the yield limit (Y).

Table 5. The results from the design point study of the frames.

	Limit state	Return period As built scantlings	Return period Required IA Super scantlings
Bow	Yield (Y)	53 days	42 days
	Plastic (P)	> life time	life time
	Ultimate (U)	> life time	> life time
Midship	Yield (Y)	43 days	60 days
	Plastic (P)	> life time	> life time
	Ultimate (U)	> life time	> life time
Stern	Yield (Y)	25 days	10 days
	Plastic (P)	7.6 years	95 days
	Ultimate (U)	12.3 years	2.6 years

4.5 Discussion

The actual yield strength of the material is higher than the nominal yield strength of 235 MPa. An estimate of the actual yield strength is about 300 MPa and this makes the return periods calculated longer. During the measurement period of years 1985 – 1991 the stress was observed several times to exceed this actual yield limit. A return period can be allocated for each case. These are presented, along with the calculated return periods of full plasticity, in Table 6. It should be noted that if the stress exceeds the actual yield limit, this does not necessarily correspond to full plasticity; thus the calculated return period is taken half way between yield and full plasticity (2 hinge formation for frames). Observed return periods are shorter for the bow structure, but for midbody and stern plating and stern frame the observed and calculated values agree quite well. Overall the comparison between calculations and observations is fair.

Table 6. The observed and calculated return period of full plasticity

Hull area	Structural element	Calculated return period ($w \leftrightarrow P$)	Observed return period
Bow	Plating	70 days	31 days
	Frame	7 years	75 days
Midbody	Plating	110 days	> 117 days
	Frame	8 years	> 140 days
Stern	Plating	50 days	63 days
	Frame	120 days	> 126 days

When observing the results in Table 6 it must be borne in mind that the response equations were calibrated using the measurements. Thus the match is not surprising for the bow structure and the relatively good match for the midbody and stern suggest that the method used i.e. determining the calibration coefficients from the frame response and from the plate response, is valid.

The calculations can be used to compare the scantlings required by the ice classes IA Super and PC6. This is done by determining the required plate thickness or frame section modulus and comparing these with those thicknesses or frame section moduli corresponding to reaching the different limit states once in ship's life time (1250 days in ice per life time). These values are presented in Table 7.

Table 7. Calculated scantlings corresponding to different limit stated once per life time compared with the (net) scantlings required by the ice class. Plastic section modulus converted to the elastic one.

Scantling	Hull area	Required by ice class		Corresponding to a limit state	
		IA Super	PC6	Y	P
t [mm]	Bow	17.4	18.0	29.9	21.2
	Midbody	13.9	11.8	22.9	16.4
	Stern	12.1	11.1	23.0	16.4
Z_e [cm^3]	Bow	713	2370	715	1170
	Midbody	412	860	347	622
	Stern	309	860	437	694

Finally, the calculations can be used to determine what scantlings are required if the return period is one year or ship's life time and the corresponding limit states yield (Y) or fully plastic response (P). The comparison is shown in Table 8. It is apparent that the as built scantlings correspond closely to the plastic limit state once per life time whereas the frames are somewhat stronger. Also these two cases are very close to each other.

Table 8. The required scantlings for two design points (Y and once per year; P and once per life time).

Limit state	Return period	Structural member	Hull area		
			Bow	Midbody	Stern
Y	1 year	Plate t [mm]	23.5	17.5	18
P	life time		21.5	16	16.5
Y	1 year	Frame Z_e [cm^3]	570	400	450
P	life time		560	360	460

5. CONCLUSION

The conclusions of this study can be divided into three different groups; the conclusions on the methods used, on the design according to ice class rules and on the loading. The methodology used in the study suggests that:

- The measurements conducted onboard MT Kemira can be used is assessing the strength with some reservations (the data is applicable to similar hull forms and operational spectrum);
- The measured ice load data should be corrected using the actual length of time in ice;
- The load statistics from shorter time periods could be used to develop conditional probability distributions for loading versus ice conditions (ice thickness);
- The plate strength formulation developed by Hayward (2001) is applicable;
- The fit between plate response measurements and calculations suggests that the flexibility coefficient is applicable;
- A comparison between measured and calculated frame response suggests that simple formulation for frame strength gives somewhat too high strength;
- Overall, the methodology developed is suitable for evaluating the different design points suggested.

The MT Kemira is used as a basis for analysing the ice class rules and also the design point. This means that the ship size dependent quantities (ice pressure) and the shell geometry (frame spacing, frame span) are those of MT Kemira. The calculations about the design point suggest that (note that the as built scantlings are used here):

- For plating, allowing once in winter yield limit response gives slightly thicker plate than allowing once in life time fully plastic response. This conclusion is valid for all hull areas. Thus using plastic design point would decrease slightly the plate thicknesses.
- A similar conclusion about the frames at the bow and midship is valid.

When comparing the scantlings (plate thickness and frame section modulus) given by the ice class IA Super and PC6, the following conclusions can be made:

- The required frame section modulus in ice class PC6 is much larger than the one required by FSICR at the bow, in other hull areas PC6 gives twice the frame requirement by IA Super;
- The plate thicknesses required for PC6 and IA Super are almost the same in all hull areas, at bow PC6 requirement is slightly higher and at midship and stern the IA Super requirement is slightly higher;
- The stern area seems to be the weakest of all the hull areas, especially for PC6;
- Using the net rule thickness, permanent plate deformation (50 % of t) at the bow, midship and stern is expected once in 4.8 years, 15.6 years and 1.1 years, respectively. The same return periods for PC6 are 6.8 years, 1.3 year and 0.4 years.

If a more thorough study about the design point when developing the ice class rules is contemplated, several aspects in this study should be improved. These include:

→ Establishing the influence of ship main particulars on the ice loading;
→ Developing a relationship between the hull shape and ice loading;

- → Collecting more ice load statistics and relating these to operational aspects;
- → Deciding upon the hull area factors based on ice loading and
- → Developing applicable frame strength formulation.

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